## **RUBRICS: HSSC ANNUAL EXAMINATION 2023 SUBJECT: MATHEMATICS HSSC-II(Local)**

Q.# /Part #	Criteria	Level 1 (Marks)	Level 2(Marks)	Level 3 (Marks)	Level 4 (Marks)	Level 5 (Marks)	Level 6 (Marks)
	Finding values of gof(x) and x for which $fog(x) = 11$	<ul> <li>(a) Correctly finding the value of <i>gof</i>(<i>x</i>).</li> <li>(2)</li> </ul>	Partially correct (1)	Wrong Answer (0)			
2( <i>i</i> )		(b) Correctly stating $f(x + 1)$ (1)	Partially correct (0.5)	Wrong Answer (0)			
		(b)Finding two correct values of $x$ . (1)	Finding one correct value of $x$ . (0.5)	Wrong Answer (0)			
	Explaining the existence of limit and discussing the	(a) Correctly finding Left Limit <b>AND</b> Right Limit at $x = 3$ . (2)	Correctly finding Left Limit <b>OR</b> Right Limit at $x = 3$ . (1)	Partially correct response (0.5)	Wrong response (0)		
2( <i>ii</i> )	continuity of the function.	(b) Correctly finding Left Limit <b>AND</b> Right Limit at $x = 12$ . (1.5)	Correctly finding Left Limit <b>OR</b> Right Limit at $x = 12$ (1)	Partially correct (0.5)	Wrong Answer (0)		
		<ul><li>(b) Correctly stating the function discontinuous.</li><li>(0.5)</li></ul>	Wrong Answer (0)				
	Showing $a\frac{dy}{dx} + btan\theta = 0$ by chain rule	Correctly finding values of $\frac{dx}{d\theta}$ AND $\frac{dy}{d\theta}$ (2)	Correctly finding value of $\frac{dx}{d\theta} \mathbf{OR} \frac{dy}{d\theta}$ (1)	Partially correct (0.5)	Wrong Answer (0)		
2(iii)		Correctly applying the chain rule <b>AND</b> Correctly showing $a\frac{dy}{dx} + btan\theta = 0$ (2)	Correctly applying the chain rule <b>OR</b> Correctly showing $a\frac{dy}{dx} + btan\theta = 0$ (1)	Partially correct (0.5)	Wrong Answer (0)		
2( <i>iv</i> )	Finding the value of $\frac{dy}{dx}$ at (1,1)	Correctly differentiating the given equation and making $\frac{dy}{dx}$ subject. (3)	Correctly differentiating the given equation not making $\frac{dy}{dx}$ subject. (2)	Partially correct (1)	Wrong Answer (0)		
2(10)		Correctly finding the value of $\frac{dy}{dx}$ at (1,1). (1)	Partially correct (0.5)	Wrong Answer (0)			

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2(11)	Showing the expansion of $cos(x + h)$ by using Taylor's series.	Correctly finding the three derivatives of <i>cosx</i> (2)	Correctly finding any two derivatives of <i>cosx</i> (1)	Correctly finding any one derivative of <i>cosx</i> (0.5)	Wrong Answer (0)		
2(v)		Correctly stating <b>AND</b> Correctly showing the Taylor's Series expansion (2).	Correctly stating <b>OR</b> Correctly showing the Taylor's Series expansion (1)	Partially correct (0.5)	Wrong Answer (0)		
	Applying second derivative test to find the conditional numbers	Correctly writing a function for extreme values <b>AND</b> Correctly finding the 1 <sup>st</sup> and 2 <sup>nd</sup> derivative of the function. (2)	Correctly writing a function for extreme values <b>OR</b> Correctly finding the 1 <sup>st</sup> and 2 <sup>nd</sup> derivative of the function. (1)	Partially correct (0.5)	Wrong Answer (0)		
2( <i>vi</i> )		Setting $1^{st}$ derivative zero and finding the correct value of $x$ (1)	Setting $1^{st}$ derivative zero and finding the incorrect value of <i>x</i> (0.5)	Wrong Answer (0)			
		Correctly applying the 2 <sup>nd</sup> derivative test and finding the correct numbers (1)	Correctly applying the 2 <sup>nd</sup> derivative test and finding the incorrect numbers (0.5)	Wrong Answer (0)			
2(vii)	Finding the maximum error in volume using differentials	Correctly stating volume of the cube <b>AND</b> Correctly finding differential of the volume function (2)	Correctly stating volume of the cube <b>OR</b> Correctly finding differential of the volume function (1)	Partially correct (0.5)	Wrong Answer (0)		
		Correctly finding the maximum error in volume. (2)	Partially correct (1)	Wrong Answer (0)			
2 (viii)	Evaluating the given integral	Correctly formulating the integral (2) Integrating correctly (2)	Partially correct (1) Partially correct (1)	Wrong Answer (0) Wrong Answer (0)			
2( <i>ix</i> )	Solving the differential equation at (2,1)	Separating the variables correctly (2) Correctly integrating LHS and RHS (1)	Partially correct (1) Integrating any of the side correctly	Wrong Answer (0) Wrong Answer (0)			
		Finding the correct solution (1)	(0.5) Partially correct (0.5)	Wrong Answer (0)			

Q.# /Part #	Criteria	Level 1 (Marks)	Level 2(Marks)	Level 3 (Marks)	Level 4 (Marks)	Level 5 (Marks)	Level 6 (Marks)
	Finding the equation of straight line with given	Correctly finding slope of the line. (2)	Partially correct (1)	Wrong Answer (0)			
2( <i>x</i> )	conditions.	Correctly formulating the line equation <b>AND</b> Correctly finding equation of the straight line (2)	Correctly formulating the line equation <b>OR</b> Correctly finding equation of the straight line (1)	Partially correct (0.5)	Wrong Answer (0)		
	Evaluating the definite integral and finding the	Integrating correctly (2)	Partially correct (1)	Wrong Answer (0)			
2( <i>xi</i> )	value of k	Correctly applying the limit <b>AND</b> correctly finding <i>k</i> 's value. (2)	Correctly applying the limit and finding the incorrect value of $k$ . (1)	Partially correct (0.5)	Wrong Answer (0)		
	Finding area of the triangle.	Correctly finding the point of intersection of the given lines. (2)	Partially correct (1)	Wrong Answer (0)			
2( <i>xii</i> )		Correctly formulating area of the triangle <b>AND</b> correctly finding area of the triangle. (2)	Correctly formulating area of the triangle <b>OR</b> correctly finding area of the triangle. (1)	Partially correct (0.5)	Wrong Answer (0)		
	Finding the equation of a circle concentric with	Correctly finding center of the circle (1)	Partially correct (0.5)	Wrong Answer (0)			
	the given circle.	Correctly finding radius of the circle (2)	Partially correct (1)	Wrong Answer (0)			
2( <i>xiii</i> )		Correctly formulating equation of the circle <b>AND</b> Correctly finding the circle equation (1)	Correctly formulating equation of the circle <b>OR</b> Correctly finding the circle equation (0.5)	Wrong Answer (0)			
	Finding height of the parabolic arch at a point 30cm from the center of the base.	Correctly writing equation of the parabolic arch. $(x^2 = 4ay)$ (1)	Partially correct (0.5)	Wrong Answer (0)			
2( <i>xiv</i> )		Correctly finding distance ' $a'$ between focus and vertex of the arch. (2)	Partially correct (1)	Wrong Answer (0)			
		Correctly finding the required height of the arch. (1)	Partially correct (0.5)	Wrong Answer (0)			

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	Verifying that vectors $\underline{a}$ and $\underline{b}$ are	Correctly finding $\underline{b} \times \underline{a}$ . (2)	Partially correct (1)	Wrong Answer (0)			
2( <i>xv</i> )	perpendicular to $\underline{b} \times \underline{a}$ .	Correctly verifying $\underline{a} \perp (\underline{b} \times \underline{a})$ AND $\underline{b} \perp (\underline{b} \times \underline{a})$ (2)	Correctly verifying $\underline{a} \perp (\underline{b} \times \underline{a}) \text{ OR } \underline{b} \perp (\underline{b} \times \underline{a})$ (1)	Partially correct (0.5)	Wrong Answer (0)		
2(xvi)	In any triangle ABC, Proving that b = c cos A + a cos C	Correctly stating $\underline{a} + \underline{b} + \underline{c} = 0$ AND Correctly stating $\underline{b} \cdot \underline{b} = (-\underline{a} - \underline{c}) \cdot \underline{b}$ (2)	Correctly stating $\underline{a} + \underline{b} + \underline{c} = 0$ <b>OR</b> Correctly stating $\underline{b} \cdot \underline{b} = (-\underline{a} - \underline{c}) \cdot \underline{b}$ (1)	Partially correct (0.5)	Wrong Answer (0)		
		Correctly verifying b = ccosA + acosC (2)	Partially correct (1)	Wrong Answer (0)			
	Finding domain and range of $f(x)$ . Finding value of k for	<ul> <li>(a) Correctly finding domain AND</li> <li>range of the function</li> <li>(1)</li> </ul>	Correctly finding domain <b>OR</b> range of the function (0.5)	Wrong Answer (0)			
3	which $f(x)$ is continuous at $x = 1$ . Stating value of $x$ when one sided limit of $f(x)$ exists. Sketching graph of f(x).	ch $f(x)$ is tinuous at $x = 1$ . ting value of $x$ en one sided limit $f(x)$ exists. tching graph of(b)Correctly applying three to conditions of continuity of the function <b>AND</b> finding the correct value of k.Correctly applying three conditions of continuity of the function <b>AND</b> finding the correct value of k.Correctly applying three conditions of continuity of the function <b>OR</b> finding the incorrect value of k.Correctly applying three conditions of continuity of the function <b>OR</b> finding the incorrect value of k.Correctly applying three conditions of continuity of the function <b>OR</b> finding the incorrect value of k.Correctly applying three conditions of continuity of the function <b>OR</b> finding the incorrect value of k.Correctly applying three of continuity of function <b>OR</b> finding the incorrect of continuity of function <b>OR</b> finding the incorrect value of k.		Correctly applying any one condition of continuity of the function <b>OR</b> finding the incorrect value of k. (1)	Wrong Answer (0)		
		<ul> <li>(c) Correctly stating the values of <i>x</i> at where Left limit AND Right limit exist.</li> <li>(2)</li> </ul>	Correctly stating the value of $x$ at where Left limit <b>OR</b> Right limit exists. (1)	Partially correct (0.5)	Wrong Answer (0)		
		<ul> <li>(d) Correctly tabulating <b>AND</b></li> <li>sketching the graph of <i>f</i>(<i>x</i>).</li> <li>(1)</li> </ul>	Correctly tabulating <b>OR</b> sketching the graph of $f(x)$ . (0.5)	Wrong Answer (0)			
4	Finding dimensions of the box having maximum volume.	<ul> <li>(a) Correctly expressing</li> <li><i>V</i> in terms of <i>x</i>, <i>h</i> AND Correctly expressing</li> <li><i>V</i> in terms of <i>x</i>.</li> <li>(2)</li> </ul>	Correctly expressing V in terms of $x$ , $h$ AND Correctly expressing V in terms of $x$ . (1)	Partially correct (0.5)	Wrong Answer (0)		

Criteria	Level 1 (Marks)	Level 2(Marks)	Level 3 (Marks)	Level 4 (Marks)	Level 5 (Marks)	Level 6 (Marks)
	(b) Correctly finding the values of $\frac{dV}{dx} \text{AND} \frac{d^2V}{dx^2}$ (2)	Correctly finding the values of $\frac{dV}{dx}$ OR $\frac{d^2V}{dx^2}$ (1)	Partially correct (0.5)	Wrong Answer (0)		
	(c) Setting $\frac{dV}{dx} = 0$ <b>AND</b> correctly finding the critical value of <i>x</i> . (1)	Setting $\frac{dV}{dx} = 0$ <b>OR</b> correctly finding the critical value of <i>x</i> . (0.5)	Wrong Answer (0)			
	<ul> <li>(c) Correctly applying the 2<sup>nd</sup></li> <li>derivative test AND finding the correct dimensions of the box.</li> <li>(2)</li> </ul>	Correctly applying the 2 <sup>nd</sup> derivative test <b>OR</b> finding the correct dimensions of the box (1)	Partially correct (0.5)	Wrong Answer (0)		
	<ul><li>(c) Correctly finding maximum volume of the box.</li><li>(1)</li></ul>	Partially correct (0.5)	Wrong Answer (0)			
Evaluating the definite integral. Sketching the graph of $f(x)$ . Shading and finding the bounded area.	<ul> <li>(a) Correctly integrating f(x) AND</li> <li>evaluating the limits.</li> <li>(2)</li> </ul>	Correctly integrating $f(x)$ <b>OR</b> evaluating the limits. (1)	Partially correct (0.5)	Wrong Answer (0)		
	(b) Correctly tabulating <b>AND</b> Correctly sketching the graph of $f(x)$ . (2)	Correctly tabulating <b>OR</b> Correctly sketching the graph of $f(x)$ . (1)	Partially correct (0.5)	Wrong Answer (0)		
	<ul> <li>(b) Correctly shading the bounded area of f(x).</li> <li>(1)</li> </ul>	Partially correct (0.5)	Wrong Answer (0)			
	<ul> <li>(c) Correctly stating the bounded area in terms of three definite integrals AND Correctly evaluating the bounded area in terms of three definite integrals.</li> <li>(3)</li> </ul>	Correctly stating the bounded area in terms of three definite integrals <b>OR</b> Evaluating partially correct bounded area in terms of three definite integrals. (2.5)	Stating partially correct bounded area in terms of three definite integrals <b>OR</b> Evaluating partially correct bounded area in terms of three definite integrals. (2)	Any one correct aspect. (1)	Wrong Answer (0)	
	integral. Sketching the graph of $f(x)$ . Shading and finding	$\frac{dv}{dx}$ AND $\frac{d^2v}{dx^2}$ (2)(c) Setting $\frac{dv}{dx} = 0$ AND correctly finding the critical value of x. (1)(c) Correctly applying the 2nd derivative test AND finding the correct dimensions of the box. (2)(c) Correctly finding maximum volume of the box. (1)Evaluating the definite integral. Sketching the graph of $f(x)$ . Shading and finding the bounded area.(a) Correctly tabulating AND Correctly sketching the graph of $f(x)$ . (2)(b) Correctly shading the bounded area of $f(x)$ . (1)(c) Correctly stating the bounded area in terms of three definite integrals AND Correctly evaluating the bounded area in terms of three definite integrals.	$\frac{d}{dx}$ AND $\frac{d}{dx^2}$ $OR \frac{d^2 v}{dx^2}$ (2)(1)(c) Setting $\frac{dv}{dx} = 0$ AND correctly finding the critical value of x. (1)Setting $\frac{dv}{dx} = 0$ OR correctly finding the critical value of x. (0.5)(i)(c) Correctly applying the $2^{nd}$ derivative test AND finding the correct dimensions of the box. (2)Correctly applying the $2^{nd}$ derivative test OR finding the correct dimensions of the box. (1)Evaluating the definite integral. Sketching the graph of $f(x)$ . Shading and finding the bounded area.(a) Correctly integrating $f(x)$ AND evaluating the limits. (2)Correctly integrating $f(x)$ OR evaluating the limits. (2)(b) Correctly tabulating AND correctly sketching the graph of $f(x)$ . (2)Correctly stating the bounded area of $f(x)$ . (1)Correctly stating the bounded area in terms of three definite integrals AND Correctly evaluating the bounded area in terms of three definite integrals AND Correctly evaluating the bounded area in terms of three definite integrals AND Correctly evaluating the bounded area in terms of three definite integrals OR Evaluating partially correct bounded area in terms of three definite integrals.	$\frac{1}{dx} \mathbf{AND} \frac{d}{dx^2} \qquad \mathbf{OR} \frac{d^2 v}{dx^2} \qquad \mathbf{OR} \frac{d^2 v}{dx^2} \qquad \mathbf{OR} \frac{d^2 v}{dx^2} \qquad \mathbf{OR} \frac{d}{dx^2} \qquad \mathbf{OR} \frac{d^2 v}{dx^2} \qquad \mathbf{OR} \frac{d^2 v}{dx^$	$\frac{\frac{1}{2x} \text{ AND } \frac{1}{2x^2}}{(2)}  OR  \frac{d^3x}{dx^2}  OR  \frac{d^3x}{dx^2}  OR  Correctly  Wrong \text{ Answer (0)}  (0.5) $	$\frac{1}{2t} \operatorname{AND} \frac{1}{dx^2} \qquad \qquad$

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	Finding vertices, area and equations of the altitudes of $\triangle ABC$ .	<ul><li>(a) Correctly finding coordinates of the vertices A, B and C.</li><li>(3)</li></ul>	Any two correct aspects. (2)	Any one correct aspect. (1)	Partially correct (0.5)	Wrong Answer (0)	
_	Showing altitudes concurrent.	(b) Correctly finding equations of the three altitudes of $\triangle ABC$ . (3)	Any two correct aspects. (2)	Any one correct aspect. (1)	Partially correct (0.5)	Wrong Answer (0)	
6		<ul> <li>(c) Correctly verifying the condition of concurrency of three straight lines.</li> <li>(1)</li> </ul>	Partially correct (0.5)	Wrong Answer (0)			
		(d)Correctly finding area of $\triangle ABC$ . (1)	Partially correct (0.5)	Wrong Answer (0)			
	Maximizing the profit function by Linear Programming	Correctly stating the profit function AND correctly finding the constraints. (2)	Correctly stating the profit function OR correctly finding the constraints. (1)	Partially correct (0.5)	Wrong Answer (0)		
		Correctly finding the intercepts. (1)	Partially correct (0.5)	Wrong Answer (0)			
7		Correctly sketching two lines with correct point of intersection and correctly shading the feasible region. (4)	Correctly sketching two lines with correct point of intersection. (3)	Correctly sketching two straight lines (2)	Correctly sketching one straight line (1)	No correct aspect (0)	
		Correctly finding the number of detailing to get maximum profit. (1)	Partially correct (0.5)	Wrong Answer (0)			
8	Finding coordinates of center, vertices, and foci; value of eccentricity and	Correctly writing the equation of horizontal hyperbola in standard form (2)	Partially correct (1)	Wrong Answer (0)			
	equations of directrices of the given hyperbola.	Correctly finding the values of a, b AND c. (1)	Correctly finding the values of a, b <b>OR</b> c. (0.5)	Wrong Answer (0)			
		Correctly finding coordinates of center, vertices, foci; value of eccentricity and equations of directrices. (5)	Any four correct aspects. (4)	Any three correct aspects. (3)	Any two correct aspects. (2)	Any one correct aspect. (1)	No correct aspect (0)

**<u>Note:</u>** All the markers must know the solutions of all the question items of the question paper before starting marking.