## RUBRICS: HSSC ANNUAL EXAMINATION 2023 <br> SUBJECT: MATHEMATICS HSSC-II(Local)

| Q.\# /Part \# | Criteria | Level 1 (Marks) | Level 2(Marks) | Level 3 (Marks) | Level 4 (Marks) | Level 5 (Marks) | Level 6 (Marks) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2(i) | Finding values of $g o f(x)$ and $x$ for which $\operatorname{fog}(x)=11$ | (a) Correctly finding the value of $g \circ f(x)$. <br> (2) | Partially correct <br> (1) | Wrong Answer (0) |  |  |  |
|  |  | (b) Correctly stating $f(x+1)$ (1) | Partially correct $(0.5)$ | Wrong Answer (0) |  |  |  |
|  |  | (b)Finding two correct values of $x$. <br> (1) | Finding one correct value of $x$. $(0.5)$ | Wrong Answer (0) |  |  |  |
| 2(ii) | Explaining the existence of limit and discussing the continuity of the function. | (a) Correctly finding Left Limit AND Right Limit at $x=3$. <br> (2) | Correctly finding Left Limit OR Right Limit at $x=3$. <br> (1) | Partially correct response (0.5) | Wrong response (0) |  |  |
|  |  | (b) Correctly finding Left Limit AND Right Limit at $x=12$. (1.5) | Correctly finding Left Limit OR Right Limit at $x=12$ <br> (1) | Partially correct $(0.5)$ | Wrong Answer (0) |  |  |
|  |  | (b) Correctly stating the function discontinuous. $(0.5)$ | Wrong Answer (0) |  |  |  |  |
| 2(iii) | Showing $a \frac{d y}{d x}+b \tan \theta=0$ by chain rule | Correctly finding values of $\frac{d x}{d \theta} \mathbf{A N D} \frac{d y}{d \theta}$ <br> (2) | Correctly finding value of $\frac{d x}{d \theta} \mathbf{O R} \frac{d y}{d \theta}$ <br> (1) | Partially correct (0.5) | Wrong Answer (0) |  |  |
|  |  | Correctly applying the chain rule AND Correctly showing $a \frac{d y}{d x}+b \tan \theta=0$ <br> (2) | Correctly applying the chain rule OR Correctly showing $a \frac{d y}{d x}+b \tan \theta=0$ <br> (1) | Partially correct $(0.5)$ | Wrong Answer (0) |  |  |
| 2(iv) | Finding the value of $\frac{d y}{d x}$ at $(1,1)$ | Correctly differentiating the given equation and making $\frac{d y}{d x}$ subject. (3) | Correctly differentiating the given equation not making $\frac{d y}{d x}$ subject. <br> (2) | Partially correct <br> (1) | Wrong Answer (0) |  |  |
|  |  | Correctly finding the value of $\frac{d y}{d x}$ at $(1,1)$. <br> (1) | Partially correct (0.5) | Wrong Answer (0) |  |  |  |


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| $2(v)$ | Showing the expansion of $\cos (x+h)$ by using Taylor's series. | Correctly finding the three derivatives of $\cos x$ <br> (2) | Correctly finding any two derivatives of $\cos x$ (1) | Correctly finding any one derivative of $\cos x$ (0.5) | Wrong Answer (0) |  |  |
|  |  | Correctly stating AND Correctly showing the Taylor's Series expansion (2). | Correctly stating OR Correctly showing the Taylor's Series expansion (1) | Partially correct (0.5) | Wrong Answer (0) |  |  |
| 2(vi) | Applying second derivative test to find the conditional numbers | Correctly writing a function for extreme values AND Correctly finding the $1^{\text {st }}$ and $2^{\text {nd }}$ derivative of the function. (2) | Correctly writing a function for extreme values OR Correctly finding the $1^{\text {st }}$ and $2^{\text {nd }}$ derivative of the function. <br> (1) | Partially correct (0.5) | Wrong Answer (0) |  |  |
|  |  | Setting $1^{\text {st }}$ derivative zero and finding the correct value of $x$ (1) | Setting $1^{\text {st }}$ derivative zero and finding the incorrect value of $x$ (0.5) | Wrong Answer (0) |  |  |  |
|  |  | Correctly applying the $2^{\text {nd }}$ derivative test and finding the correct numbers (1) | Correctly applying the $2^{\text {nd }}$ derivative test and finding the incorrect numbers (0.5) | Wrong Answer (0) |  |  |  |
| 2 (vii) | Finding the maximum error in volume using differentials | Correctly stating volume of the cube AND Correctly finding differential of the volume function (2) | Correctly stating volume of the cube OR Correctly finding differential of the volume function (1) | Partially correct (0.5) | Wrong Answer (0) |  |  |
|  |  | Correctly finding the maximum error in volume. (2) | Partially correct <br> (1) | Wrong Answer (0) |  |  |  |
| 2 (viii) | Evaluating the given integral | Correctly formulating the integral (2) | Partially correct (1) | Wrong Answer (0) |  |  |  |
|  |  | Integrating correctly (2) | Partially correct (1) | Wrong Answer (0) |  |  |  |
| 2(ix) | Solving the differential equation at $(2,1)$ | Separating the variables correctly (2) | Partially correct (1) | Wrong Answer (0) |  |  |  |
|  |  | Correctly integrating LHS and RHS (1) | Integrating any of the side correctly (0.5) | Wrong Answer (0) |  |  |  |
|  |  | Finding the correct solution (1) | Partially correct $(0.5)$ | Wrong Answer (0) |  |  |  |


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| $2(x)$ | Finding the equation of straight line with given conditions. | Correctly finding slope of the line. (2) | Partially correct (1) | Wrong Answer (0) |  |  |  |
|  |  | Correctly formulating the line equation AND Correctly finding equation of the straight line (2) | Correctly formulating the line equation OR Correctly finding equation of the straight line (1) | Partially correct (0.5) | Wrong Answer (0) |  |  |
| 2(xi) | Evaluating the definite integral and finding the value of $k$ | Integrating correctly (2) | Partially correct (1) | Wrong Answer (0) |  |  |  |
|  |  | Correctly applying the limit AND correctly finding $k$ 's value. <br> (2) | Correctly applying the limit and finding the incorrect value of $k$. (1) | Partially correct $(0.5)$ | Wrong Answer (0) |  |  |
| 2(xii) | Finding area of the triangle. | Correctly finding the point of intersection of the given lines. (2) | Partially correct (1) | Wrong Answer (0) |  |  |  |
|  |  | Correctly formulating area of the triangle AND correctly finding area of the triangle. (2) | Correctly formulating area of the triangle OR correctly finding area of the triangle. <br> (1) | Partially correct $(0.5)$ | Wrong Answer (0) |  |  |
| 2(xiii) | Finding the equation of a circle concentric with the given circle. | Correctly finding center of the circle (1) | Partially correct $(0.5)$ | Wrong Answer (0) |  |  |  |
|  |  | Correctly finding radius of the circle (2) | Partially correct (1) | Wrong Answer (0) |  |  |  |
|  |  | Correctly formulating equation of the circle AND Correctly finding the circle equation (1) | Correctly formulating equation of the circle OR Correctly finding the circle equation (0.5) | Wrong Answer (0) |  |  |  |
| 2(xiv) | Finding height of the parabolic arch at a point 30 cm from the center of the base. | Correctly writing equation of the parabolic arch. $\left(x^{2}=4 a y\right)$ <br> (1) | Partially correct (0.5) | Wrong Answer (0) |  |  |  |
|  |  | Correctly finding distance ' $\boldsymbol{a}$ ' between focus and vertex of the arch. (2) | Partially correct (1) | Wrong Answer (0) |  |  |  |
|  |  | Correctly finding the required height of the arch. <br> (1) | Partially correct (0.5) | Wrong Answer (0) |  |  |  |


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| $2(x v)$ | Verifying that vectors $\underline{a}$ and $\underline{b}$ are perpendicular to $\underline{b} \times \underline{a}$. | Correctly finding $\underline{b} \times \underline{a}$. <br> (2) | Partially correct (1) | Wrong Answer (0) |  |  |  |
|  |  | $\begin{aligned} & \text { Correctly verifying } \\ & \underline{a} \perp(\underline{b} \times \underline{a}) \text { AND } \underline{b} \perp(\underline{b} \times \underline{a}) \\ & (2) \end{aligned}$ | Correctly verifying $\underline{a} \perp(\underline{b} \times \underline{a}) \mathbf{O R} \underline{b} \perp(\underline{b} \times \underline{a})$ <br> (1) | Partially correct (0.5) | Wrong Answer (0) |  |  |
| $2(x v i)$ | In any triangle ABC , Proving that$b=c \cos A+a \cos C$ | Correctly stating $\underline{a}+\underline{b}+\underline{c}=0$ AND Correctly stating $\underline{b} \cdot \underline{b}=(-\underline{a}-\underline{c}) \cdot \underline{b}$ (2) | Correctly stating $\underline{a}+\underline{b}+\underline{c}=0 \mathbf{O R}$ Correctly stating $\underline{b} \cdot \underline{b}=(-\underline{a}-\underline{c}) \cdot \underline{b}$ (1) | Partially correct (0.5) | Wrong Answer (0) |  |  |
|  |  | Correctly verifying $b=c \cos A+a \cos C$ (2) | Partially correct (1) | Wrong Answer (0) |  |  |  |
| 3 | Finding domain and range of $f(x)$. <br> Finding value of $k$ for which $f(x)$ is continuous at $x=1$. Stating value of $x$ when one sided limit of $f(x)$ exists. Sketching graph of $f(x)$. | (a) Correctly finding domain AND range of the function <br> (1) | Correctly finding domain OR range of the function $(0.5)$ | Wrong Answer (0) |  |  |  |
|  |  | (b)Correctly applying three conditions of continuity of the function AND finding the correct value of $k$. (4) | Correctly applying three conditions of continuity of the function OR finding the incorrect value of k . (3) | Correctly applying any two conditions of continuity of the function OR finding the incorrect value of k . <br> (2) | Correctly applying any one condition of continuity of the function OR finding the incorrect value of k . (1) | Wrong <br> Answer (0) |  |
|  |  | (c) Correctly stating the values of $x$ at where Left limit AND Right limit exist. (2) | Correctly stating the value of $x$ at where Left limit OR Right limit exists. (1) | Partially correct (0.5) | Wrong Answer (0) |  |  |
|  |  | (d) Correctly tabulating AND sketching the graph of $f(x)$. (1) | Correctly tabulating OR sketching the graph of $f(x)$. (0.5) | Wrong Answer (0) |  |  |  |
| 4 | Finding dimensions of the box having maximum volume. | (a) Correctly expressing <br> $V$ in terms of $x, h \quad$ AND Correctly expressing <br> $V$ in terms of $x$. <br> (2) | Correctly expressing $V$ in terms of $x, h \quad$ AND Correctly expressing $V$ in terms of $x$. (1) | Partially correct (0.5) | Wrong Answer (0) |  |  |


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|  |  | (b) Correctly finding the values of $\frac{d V}{d x}$ AND $\frac{d^{2} V}{d x^{2}}$ <br> (2) | Correctly finding the values of $\frac{d V}{d x}$ OR $\frac{d^{2} V}{d x^{2}}$ <br> (1) | Partially correct (0.5) | Wrong Answer (0) |  |  |
|  |  | (c) Setting $\frac{d V}{d x}=0$ AND correctly finding the critical value of $x$. (1) | Setting $\frac{d V}{d x}=0$ OR correctly finding the critical value of $x$. (0.5) | Wrong Answer (0) |  |  |  |
|  |  | (c) Correctly applying the $2^{\text {nd }}$ derivative test AND finding the correct dimensions of the box. (2) | Correctly applying the $2^{\text {nd }}$ derivative test OR finding the correct dimensions of the box (1) | Partially correct (0.5) | Wrong Answer (0) |  |  |
|  |  | (c) Correctly finding maximum volume of the box. <br> (1) | Partially correct (0.5) | Wrong Answer (0) |  |  |  |
| 5 | Evaluating the definite integral. <br> Sketching the graph of $f(x)$. <br> Shading and finding the bounded area. | (a) Correctly integrating $f(x)$ AND evaluating the limits. <br> (2) | Correctly integrating $f(x)$ OR evaluating the limits. <br> (1) | Partially correct (0.5) | Wrong Answer (0) |  |  |
|  |  | (b) Correctly tabulating AND Correctly sketching the graph of $f(x)$. <br> (2) | Correctly tabulating OR Correctly sketching the graph of $f(x)$. <br> (1) | Partially correct $(0.5)$ | Wrong Answer (0) |  |  |
|  |  | (b) Correctly shading the bounded area of $f(x)$. <br> (1) | Partially correct $(0.5)$ | Wrong Answer (0) |  |  |  |
|  |  | (c) Correctly stating the bounded area in terms of three definite integrals AND Correctly evaluating the bounded area in terms of three definite integrals. <br> (3) | Correctly stating the bounded area in terms of three definite integrals OR Evaluating partially correct bounded area in terms of three definite integrals. (2.5) | Stating partially correct bounded area in terms of three definite integrals OR Evaluating partially correct bounded area in terms of three definite integrals. (2) | Any one correct aspect. <br> (1) | Wrong Answer (0) |  |


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| 6 | Finding vertices, area and equations of the altitudes of $\triangle A B C$. Showing altitudes concurrent. | (a) Correctly finding coordinates of the vertices $\mathrm{A}, \mathrm{B}$ and C . <br> (3) | Any two correct aspects. (2) | Any one correct aspect. <br> (1) | Partially correct (0.5) | Wrong Answer (0) |  |
|  |  | (b) Correctly finding equations of the three altitudes of $\triangle A B C$. <br> (3) | Any two correct aspects. (2) | Any one correct aspect. <br> (1) | Partially correct (0.5) | Wrong Answer (0) |  |
|  |  | (c) Correctly verifying the condition of concurrency of three straight lines. (1) | Partially correct $(0.5)$ | Wrong Answer (0) |  |  |  |
|  |  | (d)Correctly finding area of $\triangle A B C$. <br> (1) | Partially correct $(0.5)$ | Wrong Answer (0) |  |  |  |
| 7 | Maximizing the profit function by Linear Programming | Correctly stating the profit function AND correctly finding the constraints. (2) | Correctly stating the profit function OR correctly finding the constraints. (1) | Partially correct (0.5) | Wrong Answer (0) |  |  |
|  |  | Correctly finding the intercepts. (1) | Partially correct $(0.5)$ | Wrong Answer (0) |  |  |  |
|  |  | Correctly sketching two lines with correct point of intersection and correctly shading the feasible region. <br> (4) | Correctly sketching two lines with correct point of intersection. (3) | Correctly sketching two straight lines (2) | Correctly sketching one straight line (1) | No correct aspect <br> (0) |  |
|  |  | Correctly finding the number of detailing to get maximum profit. (1) | Partially correct (0.5) | Wrong Answer (0) |  |  |  |
| 8 | Finding coordinates of center, vertices, and foci; value of eccentricity and equations of directrices of the given hyperbola. | Correctly writing the equation of horizontal hyperbola in standard form (2) | Partially correct <br> (1) | Wrong Answer (0) |  |  |  |
|  |  | Correctly finding the values of $\mathrm{a}, \mathrm{b}$ AND c. <br> (1) | Correctly finding the values of a, b OR c. $(0.5)$ | Wrong Answer (0) |  |  |  |
|  |  | Correctly finding coordinates of center, vertices, foci; value of eccentricity and equations of directrices. (5) | Any four correct aspects. (4) | Any three correct aspects. <br> (3) | Any two correct aspects. <br> (2) | Any one correct aspect. (1) | No correct aspect (0) |

Note: All the markers must know the solutions of all the question items of the question paper before starting marking.

