Federal Board HSSC-II Examination
Mathematics Model Question Paper
(Curriculum 2002)

SECTION-A (1×20 = 20 Marks)

Answer 1 Each part carries (01) mark. Key for MCQs is as under.

i	C	ii	A	iii	A	iv	D	v	D	vi	A	vii	B	viii	A	ix	B	x	A
xi	A	xii	D	xiii	C	xiv	C	xv	C	xvi	C	xvii	B	xviii	A	xix	A	xx	C

SECTION-B (4×12 = 48 Marks)

Answer 2 Attempt any TWELVE parts. Each part carries (04) marks.

(i) $f(x) = -4 + \sqrt{3-x}\ ,\quad g(x) = \sqrt{x}$

(a) $fog(x) = f[g(x)] = f(\sqrt{x}) = -4 + \sqrt{3-\sqrt{x}}$ $(01\ mark)$

(b) $gof(x) = g[f(x)] = g(-4+\sqrt{3-x}) = \sqrt{-4+\sqrt{3-x}}$ $(01\ mark)$

(c) $fof(x) = f[f(x)] = f(-4+\sqrt{3-x}) = -4 + \sqrt{7-\sqrt{3-x}}$ $(01\ mark)$

(d) $gog(x) = g[g(x)] = g(\sqrt{x}) = \sqrt{\sqrt{x}}$ $(01\ mark)$

(ii) (a) $f(x) = -4 + \sqrt{3-x}$

Domain$(f) = (-\infty,\ 3] \ = Range(f^{-1})$ $(01\ mark)$

Range$(f) \ = [-4,+\infty) = Domain(f^{-1})$ $(01\ mark)$

(b) $f(x) = \frac{7+x}{x-1}\ ,\ x \neq 1$

Domain$(f) = \mathcal{R} - \{1\} \ = Range(f^{-1})$ $(01\ mark)$

Range$(f) \ = \mathcal{R} - \{1\} = Domain(f^{-1})$ $(01\ mark)$

(iii) $f(x) = (x^4 - x^3 + x^2 - x + 1)(3x^3 - 2x^2 + x - 1)$

Differentiating w.r.t.'x'

$f'(x) = (x^4 - x^3 + x^2 - x + 1)[9x^2 - 4x + 1] + (3x^3 - 2x^2 + x - 1)[4x^3 - 3x^2 + 2x - 1]$ $(02\ marks)$

$f'(1) = (1-1+1-1+1)[9-4+1] + (3-2+1-1)[4-3+2-1]$ $(01\ mark)$

$f'(1) = (1)[6] + (1)[2] = 8$ $(01\ mark)$

(iv) $x = 3 + cost\quad ;\qquad y = 1 - sint$

Differentiating w.r.t.'t'

$\frac{dx}{dt} = -sint \qquad \frac{dy}{dt} = -cost$ $(01+01)marks$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ $(01\ mark)$

$\frac{dy}{dx} = \frac{-cost}{-sint} = cott$ $(01\ mark)$

(v) $f(x) = (x^2 - 6x + 8)(x - 5)$

$f(x) = x^3 - 11\,x^2 + 38x - 40$

Differentiating w.r.t. x

$f'(x) = 3x^2 - 22x + 38$ $(01\ mark)$

At extreme $f'(x) = 0$

$x^2 - 22x + 38 = 0 \Longrightarrow x = \frac{22\pm\sqrt{484-456}}{6} = \frac{11\pm\sqrt{7}}{3} = 2.8\ or\ 4.5$ $(01\ mark)$

Consider $f'(2) = 3(2)^2 - 22(2) + 38 = 6 > 0 \Longrightarrow f$ increases in $(-\infty, 2.8)$

Consider $f'(3) = 3(3)^2 - 22(3) + 38 = -1 < 0 \Longrightarrow f$ decreases in $(2.8, 4.5)$

Consider $f'(5) = 3(5)^2 - 22(5) + 38 = 3 > 0 \Longrightarrow f$ increases in $(4.5, +\infty)$

f increases in $(-\infty, 2.8) \cup (4.5, +\infty)$ and decreases in $(2.8, 4.5)$ $(01+01)marks$

(vi) Let $f(x) = x^{1/5}$ with $x = 32$ and $\delta x = dx = 1$ $(01\ mark)$

Differentiating w.r.t. 'x'

$f'(x) = \frac{1}{5}x^{-4/5}$ $(01\ mark)$

$\because f(x+\delta x) = f(x) + f'(x)dx$

$f(x+\delta x) = x^{1/5} + \frac{1}{5}x^{-4/5}dx$ $(01\ mark)$

$f(32+1) = (32)^{1/5} + \frac{1}{5}(32)^{-4/5}(1)$

$f(33) = 2 + \frac{1}{80}$

$33^{1/5} = \frac{161}{80}$ $(01\ mark)$

(vii) $\int \frac{lnx}{x^2}dx = \int (lnx)(x)^{-2}\,dx$ $(01\ mark)$

$= (lnx)\left(-\frac{1}{x}\right) - \int \left(\frac{1}{x}\right)\left(-\frac{1}{x}\right)dx$ $(01\ mark)$

$= -\frac{1}{x}lnx + \int x^{-2}dx$ $(01\ mark)$

$= -\frac{1}{x}lnx - \frac{1}{x} + c$ $(01\ mark)$

(viii) $f(x) = 4x - x^2$

For x-intercepts $4x - x^2 = 0 \Longrightarrow$ $x(4-x) = 0 \Longrightarrow x = 0, 4$ $(01\ mark)$

Required Area $= \int_0^4 f(x) = \int_0^4 (4x - x^2)dx$ $(01\ mark)$

$= \left[\frac{4x^2}{2} - \frac{x^3}{3}\right]_0^4 = \frac{1}{3}\left[6x^2 - x^3\right]_0^4$ $(01\ mark)$

$= \frac{32}{3}$Square units $(01\ mark)$

(ix) Let l be the required line having slope $m = tan30° = \frac{1}{\sqrt{3}}$ $(01\ mark)$

Equation of the line l passing through $(x_1, y_1) = (-4, 8)$ with slope $m = \frac{1}{\sqrt{3}}$ is

$y - y_1 = m(x - x_1)$ $(01\ mark)$

$y - 8 = \frac{1}{\sqrt{3}}(x+4)$ $(01\ mark)$

$x - \sqrt{3}y + 8\sqrt{3} + 4 = 0$ $(01\ mark)$

(x) Let $l : 2x - 4y + 7 = 0$ be the given line

Now $-2x + 4y - 7 = 0$

At $P(3,1)$

$-2x + 4y - 7 = -2(3) + 4(1) - 7 = -9 < 0$ $(01\ mark)$

P lies below l $(01\ mark)$

At $Q(-1,6)$

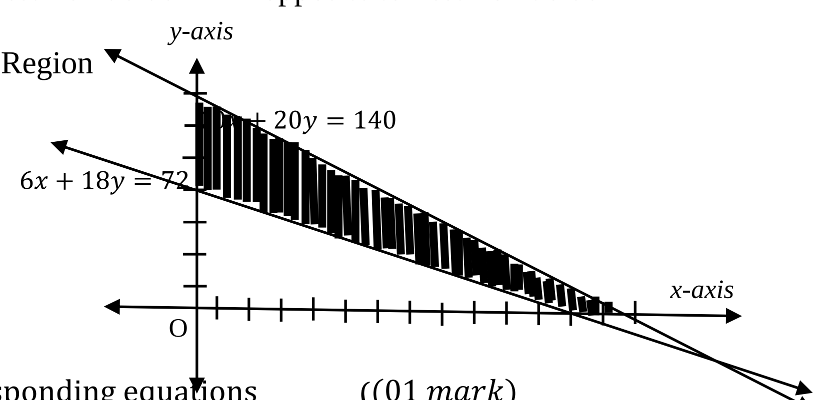
$-2x + 4y - 7 = -2(-1) + 4(6) - 7 = 20 > 0$ $(01\ mark)$

Q lies above l $(01\ mark)$

(xi)

Constraints:	$10x + 20y \leq 140$;	$6x + 18y \geq 72$; ≥ 0 ; $y \geq 0$	
Corresponding Equations:	$10x + 20y = 140$;	$6x + 18y = 72$	
Intercepts:	$(14,0), (0,7)$	$(12,0), (0,4)$	$(01\ mark)$
In Equations:	$10x + 20y < 140$;	$6x + 18y > 72$	
Test Point: $O(0,0)$	$0 < 140$	$0 > 72$	$(01\ mark)$
Solution Region lies	towards Test Point side	opposite to Test Point side	

Common shaded region is the Solution Region



Graph: $\begin{cases}\text{Lines representing the corresponding equations} \\ \text{Correct shade of the feasible solution region}\end{cases}$ $\begin{cases}(01\ mark) \\ (01\ mark)\end{cases}$

(xii) Let $l_1 : 3y = 4x - 5$ and $l_2 : 3y = -4x - 13$

$l_1 + l_2$: $6y = -18$ $\Longrightarrow y = -3$

$l_1 - l_2$: $0 = 8x + 8$ $\Longrightarrow x = -1$

Centre of the circle: $C(h,k) = (-1,-3)$ $(01\ mark)$

If$P(-5,0)$ be the point lying on the circle,

then radius $r = |CP| = \sqrt{(-5+1)^2 + (0+3)^2} = 5$ $(01\ mark)$

Circle Equation: $(x-h)^2 + (y-k)^2 = r^2$ $(01\ mark)$

$(x+1)^2 + (y+3)^2 = 5^2$

or $x^2 + y^2 + 2x + 6y - 15 = 0$ $(01\ mark)$

(xiii) Let $F(-2,1)$ be the focus and $l: x-5=0$ be the directrix of the parabola.
Consider a point $P(x,y)$ on the parabola. Draw $\overline{PM} \perp l$ and join P to F.
By definition $|\overline{PF}| = |\overline{PM}|$ $(01\ mark)$

$\sqrt{(x+2)^2+(y-1)^2} = \frac{|x-5|}{\sqrt{1^2}}$ $(01\ mark)$

$(x+2)^2+(y-1)^2 = (x-5)^2$ $(01\ mark)$

$(y-1)^2 = 7(3-2x)$ $(01\ mark)$

(xiv) $16x^2+25y^2 = 1 \Rightarrow \frac{x^2}{1/16}+\frac{y^2}{1/25} = 1$

Comparing with $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ to get $a^2 = 1/16$ and $b^2 = 1/25$

At the point $(x_1,y_1) = (4, 12/5)$

Equation of the tangent: $\frac{xx_1}{a^2}+\frac{yy_1}{b^2} = 1$ $(01\ mark)$

$\Rightarrow \frac{4x}{1/16}+\frac{12y/5}{1/25} = 1 \Rightarrow 64x+60y-1 = 0$ $(01\ mark)$

Equation of the normal: $y-y_1 = \frac{a^2y_1}{b^2x_1}(x-x_1)$ $(01\ mark)$

$\Rightarrow y-\frac{12}{5} = \frac{\left(\frac{1}{16}\right)\left(\frac{12}{5}\right)}{\left(\frac{1}{25}\right)4}(x-4) \Rightarrow 75x-80y-108 = 0$ $(01\ mark)$

(xv) $\underline{u} = -2\underline{i}+5\underline{j}+3\underline{k}$; $\underline{v} = \underline{i}+3\underline{j}-2\underline{k}$; $\underline{w} = -3\underline{i}+\underline{j}-2\underline{k}$

Volume of Parallelepiped $= \begin{vmatrix} -2 & 5 & 3 \\ 1 & 3 & -2 \\ -3 & 1 & -2 \end{vmatrix}$ $(02\ marks)$

$= -2(-4)-5(-8)+3(10) = 78$ $(01+01) marks$

(xvi) $\underline{u} = 3\underline{i}+\underline{j}-\underline{k}$; $\underline{v} = 2\underline{i}-\underline{j}+\underline{k}$

$\underline{u}.\underline{v} = (3)(2)+(1)(-1)+(-1)(1) = 4$ $(01\ mark)$

$|\underline{u}| = \sqrt{9+1+1} = \sqrt{11}$; $|\underline{v}| = \sqrt{4+1+1} = \sqrt{6}$ $(01\ mark)$

If θ be the angle between $\underline{u}$ and $\underline{v}$, then

$\cos\theta = \frac{\underline{u}.\underline{v}}{|\underline{u}||\underline{v}|} = \frac{4}{\sqrt{11}\sqrt{6}}$ $(01\ mark)$

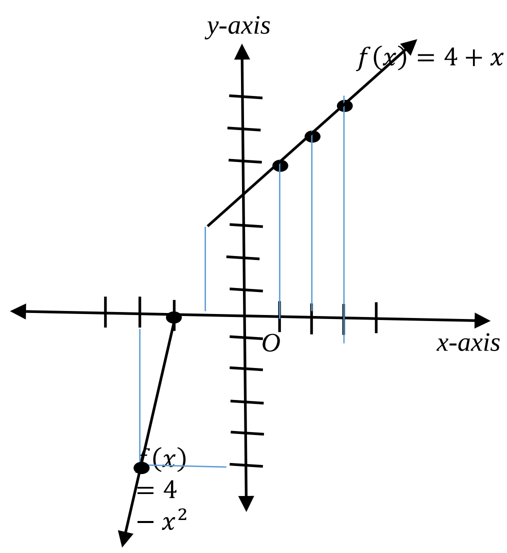
$\theta = \cos^{-1}\left(\frac{4}{\sqrt{66}}\right) = 60.5°$ $(01\ mark)$

SECTION-C (8×4 = 32 Marks)

Answer 3 $f(x) = \begin{cases} 4-x^2, & if\ x \le 0 \\ 4+x, & if\ x > 0 \end{cases}$

(a) Correct Table of values $(01\ mark)$
Correct graph $(01\ mark)$

x	-3	-2	-1	0	1	2	3
y	-5	0	3	4	5	6	7



(b) At $x = 0$, $f(x) = 4-x^2$
$\Rightarrow f(0) = 4$ $(01\ mark)$

(c) $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}(4-x^2) = 4$ $(01\ mark)$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}(4+x) = 4$ $(01\ mark)$

(d) Since $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = 4$

$\Rightarrow \lim_{x\to 0} f(x) = 4$ $(01\ mark)$

And $\lim_{x\to 0} f(x) = f(0)$ $(01\ mark)$

Therefore f is continuous at $x = 0$ $(01\ mark)$

Answer 4 $f(x) = sinx+\cos^2 x$; $x\epsilon\left[0,\frac{\pi}{2}\right]$

(a) Differentiating w.r.t. x

$f'(x) = cosx-2cosx.sinx$ $(01\ mark)$

(b) Differentiating again w.r.t. x

$f''(x) = -sinx-2(\cos^2 x-\sin^2 x) = -sinx-2cos2x$ $(01\ mark)$

(c) For extreme values put $f'(x) = 0$

$cosx - 2cosx.sinx = 0 \Longrightarrow cosx(1 - 2sinx) = 0$

$cosx = 0$ gives $x = \frac{\pi}{2}$ $(01\ mark)$

$1 - 2sinx = 0$ gives $sinx = \frac{1}{2} \Longrightarrow x = \frac{\pi}{6}$ $(01\ mark)$

(d) $f''\left(\frac{\pi}{2}\right) = -sin\left(\frac{\pi}{2}\right) - 2cos2\left(\frac{\pi}{2}\right) = 1 > 0$ $(01\ mark)$

$f_{min} = f\left(\frac{\pi}{2}\right) = sin\left(\frac{\pi}{2}\right) + \cos^2\left(\frac{\pi}{2}\right) = 1$ $(01\ mark)$

$f''\left(\frac{\pi}{6}\right) = -sin\left(\frac{\pi}{6}\right) - 2cos2\left(\frac{\pi}{6}\right) = -\frac{3}{2} < 0$ $(01\ mark)$

$f_{max} = f\left(\frac{\pi}{6}\right) = sin\left(\frac{\pi}{6}\right) + \cos^2\left(\frac{\pi}{6}\right) = \frac{5}{4}$ $(01\ mark)$

Answer 5 $\int \frac{x^3+4}{(x^2-1)(x^2+3x+2)}dx$

(a) $\frac{x^3+4}{(x+1)^2(x-1)(x+2)} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x-1)} + \frac{D}{(x+2)} \rightarrow eqn\ I$

$x^3 + 4 = A(x+1)(x-1)(x+2) + B(x-1)(x+2) + C(x+1)^2(x+2) + D(x+1)^2(x-1) \rightarrow eqn\ II$

For B put $x = -1$ $\quad -1 + 4 = B(-2)(1) \Longrightarrow B = -3/2$ $(01\ mark)$

For C put $x = 1$ $\quad 1 + 4 = C(4)(3) \quad \Longrightarrow C = 5/12$ $(01\ mark)$

For D put $x = -2$ $\quad -8 + 4 = D(1)(-3) \Longrightarrow D = 4/3$ $(01\ mark)$

Consider $eqn\ II$ as

$x^3 + 4 = A(x^3 + 2x^2 - x - 2) + B(x^2 + x - 2) + C(x^3 + 4x^2 + 5x + 2) + D(x^3 + x^2 - x - 1)$

Equating the coefficients of like powers of x^3

$1 = A + C + D$

$1 = A + \frac{5}{12} + \frac{4}{3} \Longrightarrow A = -\frac{3}{4}$ $(01\ mark)$

Substituting values in $eqn\ I$

$\frac{x^3+4}{(x+1)^2(x-1)(x+2)} = -\frac{3}{4(x+1)} - \frac{3}{2(x+1)^2} + \frac{5}{12(x-1)} + \frac{4}{3(x+2)}$

(b) $\int \frac{x^3+4}{(x+1)^2(x-1)(x+2)}dx = -\frac{3}{4}\int \frac{1}{(x+1)}dx - \frac{3}{2}\int (x+1)^{-2}dx + \frac{5}{12}\int \frac{1}{(x-1)}dx + \frac{4}{3}\int \frac{1}{(x+2)}dx$

$\int \frac{x^3+4}{(x+1)^2(x-1)(x+2)}dx = -\frac{3}{4}ln(x+1) + \frac{3}{2(x+1)} + \frac{5}{12}ln(x-1) + \frac{4}{3}ln(x+2)$

$(01 + 01 + 01 + 01)marks$

Answer 6 $A(-1, 1), B(5, 5), C(4, 1)$

(a) Slope of $l = m = \frac{5-1}{5+1} = \frac{2}{3}$ $(01\ mark)$

(b) Equation of l through point $(x_1, y_1) = A(-1,\ 1)$

is given by $\quad y - y_1 = m(x - x_1)$ $(01\ mark)$

$y - 1 = \frac{2}{3}(x + 1) \Longrightarrow \quad 2x - 3y + 5 = 0$ $(01\ mark)$

(c) $2x - 3y + 5 = 0 \Longrightarrow -2x + 3y - 5 = 0$

Divide the eqn by $\sqrt{(2)^2 + (-3)^2}$ or $\sqrt{13}$

$-\frac{2}{\sqrt{13}}x + \frac{3}{\sqrt{13}}y = \frac{5}{\sqrt{13}}$ $(01\ mark)$

Comparing with $xcos\alpha + ysin\alpha = p$

$cos\alpha = -\frac{2}{\sqrt{13}} < 0$; $sin\alpha = \frac{3}{\sqrt{13}} > 0$; $p = \frac{5}{\sqrt{13}}$ (α in 2nd Quadrant)

$\Longrightarrow tan\alpha = -\frac{3}{2}\alpha = \tan^{-1}\left(-\frac{3}{2}\right) = 123.69°$ $(01\ mark)$

Normal Form: $xcos(123.69°) + ysin(123.69°) = \frac{5}{\sqrt{13}}$ $(01\ mark)$

(d) $A(-1, 1),\ B(5, 5),\ C(4, 1)$

Area of Triangle ABC $= \frac{1}{2}\begin{vmatrix} -1 & 1 & 1 \\ 4 & 1 & 1 \\ 5 & 5 & 1 \end{vmatrix}$ $(01\ mark)$

$= \frac{1}{2}[-1(-4) - 1(-1) + 1(15)] = 10$ $(01\ mark)$

Answer 7 Let number of conventional phones be x and number of smart phones be y.

Time constraints for assembling and finishing of conventional and smart phones are $x + 2y \le 24$ and $2x + y \le 24$ respectively.

The restriction on number of gadgets in a day is $x + y \le 15$

The objective function as Profit function is $P(x, y) = 1000x + 4000y$

Constraints: $x + 2y \le 24$; $2x + y \le 24$; ≥ 0 ; $y \ge 0$

Corresponding Equations: $x + 2y = 24$; $2x + y = 24$

Intercepts: $(24, 0), (0, 12)$; $(12, 0), (0, 24)$ $(01\ mark)$

In Equations: $x + 2y < 24$; $2x + y < 24$

Test Point: $O(0, 0)$ $\quad 0 < 24; 0 < 24$ $(01\ mark)$

Solution Region lies: towards Test Point side

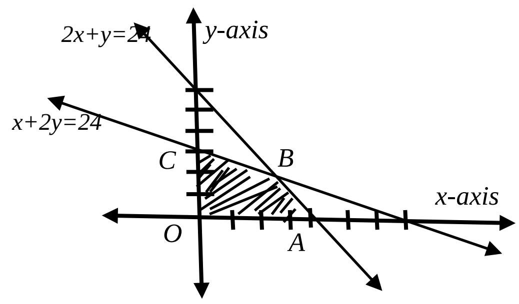
The graph shows OABC as feasible solution region.

For B solving $x + 2y = 24$ and $2x + y = 24$

$\frac{x}{-48+24} = \frac{-y}{-24+48} = \frac{1}{1-4} \Longrightarrow x = 8,\ y = 8 \Longrightarrow B(8,8)$ $(01\ mark)$

Graph: $\begin{cases} \text{Lines representing the corresponding equations} \\ \text{Correct shade of the feasible solution region} \end{cases}$ $\begin{cases} (01 + 01) marks \\ (01\ mark) \end{cases}$

Corner Points	Objective Function $P(x, y) = 1000x + 4000y$
$O(0,0)$	$P(0,0) = 0$
$A(12,0)$	$P(12,0) = 12{,}000$
$B(8,8)$	$P(8,8) = 40{,}000$
$C(0,12)$	$P(0,12) = 48{,}000$ (max)



Correct Table of values $(01\ mark)$

The point C(0, 12) gives the most profit, and that profit is Rs. 48,000.
Therefore, we conclude that one should manufacture 12 smart phones daily to obtain the maximum profit. $(01\ mark)$

Answer 8 $4x^2 - 5y^2 + 40x - 30y - 45 = 0$

$4(x+5)^2 - 5(y+3)^2 = 100$

$\frac{(x+5)^2}{25} - \frac{(y+3)^2}{20} = 1$ (Horizontal Hyperbola) $(01\ mark)$

$\frac{X^2}{25} - \frac{Y^2}{20} = 1$ where $X = x + 5$ and $Y = y + 3$

Comparing with $\frac{X^2}{a^2} - \frac{Y^2}{b^2} = 1$ to get $a = 5$ and $b = 2\sqrt{5}$ $(01\ mark)$

Taking $c^2 = a^2 + b^2 = 25 + 20 = 45 \Longrightarrow c = 3\sqrt{5}$ $(01\ mark)$

Centre: $(0,0) \Longrightarrow X = 0 \quad ; Y = 0$

$\Longrightarrow x + 5 = 0\ ;\ y + 3 = 0$

$\Longrightarrow x = -5\ ;\ y = -3 \Longrightarrow (-5,-3)$ $(01\ mark)$

Foci: $(\pm c, 0) \Longrightarrow X = \pm c \quad ; Y = 0$

$\Longrightarrow x + 5 = \pm 3\sqrt{5}\ ;\ y + 3 = 0$

$\Longrightarrow x = -5 \pm 3\sqrt{5}\ ;\ y = -3 \Longrightarrow \left(-5 \pm 3\sqrt{5},\ -3\right)$ $(01\ mark)$

Eccentricity: $e = \frac{c}{a} = \frac{3\sqrt{5}}{5}$ $(01\ mark)$

Vertices :$(\pm a,\ 0) \Longrightarrow X = \pm a\ ;\ Y = 0$

$\Longrightarrow x + 5 = \pm 5\ ;\ y + 3 = 0$

$\Longrightarrow x = -5 \pm 5\ ;\ y = -3 \Longrightarrow (-5 \pm 5,\ -3)$ $(01\ mark)$

Directrices : $X = \pm \frac{c}{e^2}$

$\Longrightarrow x + 5 = \pm \frac{3\sqrt{5}}{\left(3\sqrt{5}/5\right)^2} \qquad \Longrightarrow x = \frac{-15 \pm 5\sqrt{5}}{3}$ $(01\ mark)$