Version No.			

ROLL NUMBER						

Answer Sheet No._____

Sign. of Candidate_____

Sign. of Invigilator_____

MATHEMATICS HSSC–II

SECTION – A (Marks 20)

Time allowed: 25 Minutes

Section – A is compulsory. All parts of this section are to be answered on this page and handed over to the Centre Superintendent. Deleting/overwriting is not allowed. **Do not use lead pencil.**

Q.1 Fill the relevant bubble for each part. All parts carry one mark.

1. What result occurs, in evaluating $\lim_{x\to 3}\frac{x^3-27}{x-3}$?

A. 9 ◯ B. -9 ◯

C. 27 ◯ D. does not exist ◯

2. Which one of the following represents an Odd function?

A. $f(x)=\frac{3x}{x^2+1}$ ◯ B. $f(x)=3x^4-2x^2+7$ ◯

C. $f(x)=sinx+cosx$ ◯ D. $f(x)=(x+2)^2$ ◯

3. Which one of the following represents $f^{-1}(\sqrt{2})$, if $f(x)=\sqrt{2\tan x}$?

A. $\frac{\pi}{4}$ ◯ B. $\frac{7\pi}{20}$ ◯

C. $\frac{\pi}{2}$ ◯ D. $\frac{3\pi}{4}$ ◯

4. If $f(x)=\cos x,\ \ x\in\left(\frac{\pi}{2},\pi\right)$ then what is the result of $f'\left(\frac{3\pi}{4}\right)$?

A. $\frac{\sqrt{3}}{2}$ ◯ B. $\frac{1}{\sqrt{2}}$ ◯

C. $-\frac{\sqrt{3}}{2}$ ◯ D. $-\frac{1}{\sqrt{2}}$ ◯

5. In which one of the following intervals, $f(x)=2x^2-8x+1$ increases its value?

A. $(-\infty,2]$ ◯ B. $(-\infty,0]$ ◯

C. $[0,\infty)$ ◯ D. $(2,\infty)$ ◯

6. For a function $f(x)=\sin(\sin x)$ what evaluates$f'(0)$?

A. 1 ◯ B. 0 ◯

C. – 1 ◯ D. does not exist ◯

7. Which one of the following options represents $f'(x)=e^x+\sin x+1$ and $f(0)=2$?

A. $f(x)=e^x+\cos x+x$ ◯

B. $f(x)=e^x-\cos x+x+2$ ◯

C. $f(x)=xe^{x-1}-\cos x+x+3$ ◯

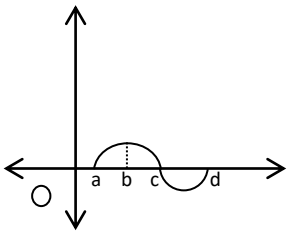
D. $f(x)=e^x+\cos x$ ◯

8. What results $\int_0^{\pi/4} \frac{e^{\tan x}}{\cos^2 x}\,dx$?

A. $e - 1$ ◯ B. e ◯

C. $\frac{\pi}{4}$ ◯ D. 0 ◯

9. The graph of $f(x) = \int_a^x g(t)dt$ is shown in the given figure. For what value of x, $f(x)$ has its maximum value?



A. a ◯ B. b ◯

C. c ◯ D. d ◯

10. Which one of the following lines passes through (–7, 7), (–7, –7) and (–7, 0)?

A. $x = -7$ ◯ B. $y = -7$ ◯

C. $x + y = -7$ ◯ D. $y = -x + 7$ ◯

11. How many intercepts are there in the graph of $y = \frac{1}{x}$?

A. no intercepts ◯ B. two *x*–intercepts ◯

C. two *y*–intercepts ◯ D. one *x* and one *y*–intercept ◯

12. At what angle lines $3y = 2x + 5$ and $3x + 2y = 8$ cut each other?

A. $\frac{\pi}{6}$ ◯ B. $\frac{\pi}{4}$ ◯

C. 0 ◯ D. $\frac{\pi}{2}$ ◯

13. Which one of the following options does not satisfy $4x - 3y < 2$?

A. $(1, 1)$ ◯ B. $(0, 0)$ ◯

C. $(3, 0)$ ◯ D. $(-2, 1)$ ◯

14. What are the coordinates of the centre of a circle $x^2 + y^2 - 8x + 12y + 21 = 0$?

A. $(4, 6)$ ◯ B. $(-4, 6)$ ◯

C. $(4, -6)$ ◯ D. $(-4, -6)$ ◯

15. What is the equation of the axis of a parabola $y^2 - 2y + 8x - 23 = 0$?

A. $y = -1$ ◯ B. $x = 3$ ◯

C. $y = 1$ ◯ D. $x = -3$ ◯

16. If (5, –2), (5, 4) are the vertices of a hyperbola, then centre of the hyperbola is:

A. (0, 0) ◯ B. (5, 3) ◯

C. (5, 1) ◯ D. (5, 0) ◯

17. Which one of the following represents the graph of $4x^2 + y^2 - 8x + 4y - 9 = 0$?

A. circle ◯ B. ellipse ◯

C. parabola ◯ D. hyperbola ◯

18. For what value of $\propto$, vectors $4\underline{i} + 3\underline{j} - 3\underline{k}$ and $\propto \underline{i} + 3\underline{k}$ have the same magnitude?

A. ± 5 ◯ B. 5 ◯

C. 25 ◯ D. -5 ◯

19. If vectors $3\underline{i} - 6\underline{j} + \underline{k}$ and $2\underline{i} - 4\underline{j} + \lambda\underline{k}$ are parallel to each other, then the value of λ is:

A. $\frac{2}{3}$ ◯ B. $\frac{3}{2}$ ◯

C. $-\frac{3}{2}$ ◯ D. $-\frac{2}{3}$ ◯

20. What is the projection of $\underline{i} - \underline{k}$ along $\underline{j} + \underline{k}$?

A. $\frac{1}{\sqrt{2}}$ ◯ B. $-\frac{1}{2}$ ◯

C. $-\frac{1}{\sqrt{2}}$ ◯ D. -1 ◯

Federal Board HSSC-II Examination
Mathematics Model Question Paper
(Curriculum 2000)

Time allowed: 2.35 hours Total Marks: 80

Note: Attempt any twelve parts from section 'B' and any four questions from 'C'. Questions therein are to be answered on the separately provided Answer Book. Write your answers neatly and legibly.

SECTION – B (Marks 48)

Q2. Attempt any **TWELVE** parts. All parts carry equal marks. (12 × 4 = 48)

i. If $f(x) = -4 + \sqrt{3-x}$ and $g(x) = \sqrt{x}$, then find the following:
$(a)\, fog(x)$ $(b)\ \ gof(x)$ $(c)\, fof(x)$ $(d)\, gog(x)$

ii. State the domain and range of f^{-1} of the function:
(i) $f(x) = -4 + \sqrt{3-x}$ (ii) $f(x) = \frac{7+x}{x-1}$, $x \neq 1$

iii. Let $f(x) = (x^4 - x^3 + x^2 - x + 1)(3x^3 - 2x^2 + x - 1)$.Use the rule for differentiating products to find$f'(1)$.

iv. Find $\frac{dy}{dx}$ if$x = 3 + cost$ and $y = 1 - sint$

v. In which interval a function $f(x) = (x^2 - 6x + 8)(x - 5)$ increases or decreases?

vi. Use differentials to approximate the value of $(33)^{1/5}$

vii. Evaluate $\int \frac{lnx}{x^2} dx$

viii. Find the area in the first quadrant bounded by $f(x) = 4x - x^2$ and the x-axis.

ix. A straight line passes through the point $(-4, 8)$ and makes an angle $30°$ with $x^+ - axis$. Find the equation of the straight line.

x. Check whether the points (3, 1) and (–1, 6) lie above or below the line $2x - 4y + 7 = 0$.

xi. Graph the solution region of the following system of linear inequalities by shading.$10x + 20y \leq 140$; $6x + 18y \geq 72; x \geq 0; y \geq 0$

xii. What is the equation of a circle when lines $3y = 4x - 5$ and $3y = -4x - 13$ are the diameters and a point (–5, 0) lies on the circle?

xiii. Write the equation of parabola with focus $(-2, 1)$and directrix $x = 5$.

xiv. Find the equations of tangent and normal to the ellipse $16x^2 + 25y^2 = 1$ at$\left(4, \frac{12}{5}\right)$.

xv. Find the volume of a parallelepiped determined by the vectors $\underline{u} = -2\underline{i} + 5\underline{j} + 3\underline{k}$, $\underline{v} = \underline{i} + 3\underline{j} - 2\underline{k}$ and $\underline{w} = -3\underline{i} + \underline{j} - 2\underline{k}$.

xvi. Find the angle between the vectors $\underline{u} = 3\underline{i} + \underline{j} - \underline{k}$ and $\underline{v} = 2\underline{i} - \underline{j} + \underline{k}$.

SECTION – C (Marks 32)

Note: Attempt any **FOUR** questions. All questions carry equal marks. $(4 \times 8 = 32)$

Q3. Let $f(x) = \begin{cases} 4 - x^2 \; if x \leq 0 \\ 4 + x \;\; if x > 0 \end{cases}$

(a) Sketch the graph of the function about $x = 0$

(b) Find the value of $f(x)$ at $x = 0$

(c) Find $\lim\limits_{x \to 0^-} f(x)$ and $\lim\limits_{x \to 0^+} f(x)$

(d) Justify the continuity/discontinuity of $f(x)$ at $x = 0$.

Q.4 Consider the function $f(x) = \sin x + \cos^2 x$, where $x \in \left[0, \frac{\pi}{2}\right]$

(a) Find $f'(x)$

(b) Find $f''(x)$

(c) Find the values of $x \in \left[0, \frac{\pi}{2}\right]$ for which $f(x)$ has maximum or minimum values.

(d) Find possible extreme values of $f(x)$

Q5. For $\int \frac{x^3+4}{(x^2-1)(x^2+3x+2)} dx$

(a) Resolve $\frac{1}{(x+1)^2(x-1)(x+2)}$ into partial fractions

(b) Evaluate the integral $\int \frac{x^3+4}{(x^2-1)(x^2+3x+2)} dx$

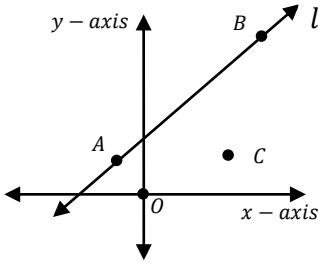
Q6. The diagram shows a line l passing through $A(-1, 1), B(5, 5)$ given that $C(4, 1)$.

(a) Find the slope of line l

(b) Find an equation of line l

(c) Write an equation of line l in normal form

(d) Find area of ΔABC



Q7. A factory manufactures two types of cell phones, conventional and smartphone.Each cell phone requires the use of two operations assembling and finishing, and there are at most 24 hours available for each operation. A conventional phone requires 1 hour of assembling and 2 hours of finishing, while a smartphone needs 2 hours of assembling and 1 hour of finishing. Due to some restrictions, the company can make at the most 15 gadgets a day. If a profit of Rs.1000 is realized for each conventional phone and Rs.4000 for a smartphone, how many of each should be manufactured to maximize the profit?

Q8. Find the centre, foci, eccentricity, vertices and equations of directrices of the conic
$4x^2 - 5y^2 + 40x - 30y - 45 = 0$

MATHEMATICS HSSC-II

Student Learning Outcomes Alignment Chart

(National Curriculum 2000)

Sec-A	Q 1	Contents and Scope	Student Learning Outcomes
	(1)	Limits of Important Functions	(a) Limit of the following functions at $x = a$ $\frac{x^n - a^n}{x-a}, \frac{x-a}{\sqrt{x}-\sqrt{a}}$
	(2)	Kinds of Functions	To know the following types of functions: Algebraic, trigonometric, inverse trigonometric, hyperbolic, explicity and implicity defined functions, parametric representation of functions, even and odd functions.
	(3)	Composition and Inversion of Functions	To know the meaning of the identity and constant functions and the techniques of composing the functions by algebraic methods.
	(4)	Differentiation of Functions other than Algebraic	To find the derivatives of trigonometric, inverse trigonometric, exponential, logarithmic, hyperbolic and inverse hyperbolic functions using chain and other rules
	(5)	Extreme Values	To find whether a function is increasing or decreasing in an interval.
	(6)	Differentiation of Functions other than algebraic	To find the derivatives of trigonometric, inverse trigonometric, exponential, logarithmic, hyperbolic and inverse hyperbolic functions using chain and other rules.
	(7)	Introduction to Integration	To define integration as anti- derivative and to know simple standard integrals which directly follow from standard differentiation formulas and to apply them in the integration of functions of the same types.
	(8)	Definite Integrals	To be able to differentiate between definite and indefinite integrals and to know and apply the theorems of definite integrals.
	(9)	Extreme Values	To have the concept of maximum and minimum values and critical points of a function.
	(10)	Equations of Straight lines	To know the position of a point with respect to a line and to find the distance of a point from a line and between two parallel lines.
	(11)	Equations of Straight lines	(d) Derivation of the following standard forms of the equations of the straight lines; slope intercept; point slope; two points; intercepts; normal and symmetric.
	(12)	Two or Three Straight lines	Be able to find: c) acute angle between two straight lines, condition of their parallelism and perpendicularity.
	(13)	Linear Inequalities and their Graphs	To know the meanings of linear inequalities in two variables and their solutions be graphically illustrated.

	(14)	Circle	(b) To know the general form of the equation of circle $x^2 + y^2 + 2gx + 2fy + c = 0$and be able to find its centre and radius.
	(15)	Parabola and its Elements	To derive the standard forms of equations of parabolas and to draw their graphs and to find the elements.
	(16)	Hyperbola and its Elements	To know the concept of a hyperbola and its elements (centre, foci, eccentricity, focal chord, latera recta, directrices, transverse and conjugate axes).
	(17)	Ellipse and its Elements	To know the concept of an ellipse and its elements (centre, foci, eccentricity, vertices, major and minor axes, focal chord, latera recta, directrices).
	(18)	Introduction of a Vector in Space	To know location of a point in space using Cartesian system; concept of vectors in space; fundamental unit vectors $\underline{i}$, $\underline{j}$, $\underline{k}$ components of a vector $\underline{a} = a_1\underline{i} + a_2\underline{j} + a_3\underline{k}$ magnitude of a vector, unit vector, parallel, collinear and coplanar vectors.
	(19)	Introduction of a Vector in Space.	To know location of a point in space using Cartesian system; concept of vectors in space; fundamental unit vectors $\underline{i}$, $\underline{j}$, $\underline{k}$ components of a vector $\underline{a} = a_1\underline{i} + a_2j + a_3\underline{k}$ magnitude of a vector, unit vector, parallel, collinear and coplanar vectors.
	(20)	Scalar Product of two Vectors	To know analytic expression of $\bar{a}.\bar{b}$i.e. if $\underline{a} = a_1\underline{i} + a_2\underline{j} + a_3\underline{k}$ and $\underline{b} = b_1\underline{i} + b_2\underline{j} + b_3\underline{k}$ then $\underline{a}.\underline{b} = a_1b_1 + a_2b_2 + a_3b_3$ angle between two vectors; projection of one vector on another vector; properties of scalar product (parallel and perpendicular vectors)
Sec-B	**Q 2-**i	Composition and Inversion of Functions	To know the meaning of the identity and constant functions and the techniques of composing the functions by algebraic methods.
	ii	Continuous and Discontinuous Functions	To understand the concept of continuity of a function at a point and in an interval intuitively, explanation of continuity and discontinuity through graphs. ,
	iii	Theorems on Differentiation	To establish the theorems on differentiation sum, difference, product and quotient of functions and their application, differentials of $y = (ax + b)^n$ where n is a negative integer, using quotient theorem.
	iv	The Chain Rule	Explanation and application of chain rule for composite functions and functions defined by parametric functions.
	v	Extreme values	To find whether a function is increasing or

			decreasing at a point and in an interval.
	vi	Differentials	Simple application of differentials in finding the approximate values of irrational numbers and $sinx, cosx,$ when $x = 29°, 46°, 62°,$ etc.
	vii	Integration by Parts	To know and be able to find the anti-derivatives of functions by parts including the standard forms.
	viii	Application of Definite Integrals	To be able to calculate areas bounded by the curve and $x - axis$.
	ix	Equations of Straight lines	To know the position of a point with respect to a line and to find the distance of a point from a line and between two parallel lines.
	x	Equations of straight lines	g) To know the position of a point with respect to a line and to find the distance of a point from a line and between two parallel lines.
	xi	Linear inequalities and their Graphs	To determine graphically the region bounded by two or three simultaneous inequalities of non-negative variables and shading the regions bounded by them.
	xii	Circle	To know the definition of a circle
	xiii	Equation of a Parabola with given elements	To find the equation of a parabola with the following given elements. • focus and vertex • focus and directrix • vertex and directrix
	xiv	Tangents and Normals to an ellipse	(c) To find the equations of tangent and normal to an ellipse at a point.
	xv	Scalar Triple Product of Vectors	(d) To find the volume of a parallelepiped and regular tetrahedron.
	xvi	Scalar Triple Product of Vectors	To know analytic expression of $\bar{a}.\bar{b}$i.e. if $\underline{a} = a_1\underline{i} + a_2\underline{j} + a_3\underline{k}$ and $\underline{b} = b_1\underline{i} + b_2\underline{j} + b_3\underline{k}$ then $\underline{a}.\underline{b} = a_1b_1 + a_2b_2 + a_3b_3$ angle between two vectors; projection of one vector on another vector; properties of scalar product (parallel and perpendicular vectors)
Sec-C	**Q No.** 3	Continuous and Discontinuous Functions	To understand the concept of continuity of a function at a point and in an interval intuitively, explanation of continuity and discontinuity through graphs.
	4	Extreme Values	• To have the concept of maximum and minimum values and critical points of a function. • To know the second derivative test of maxima and minima.
	5	Integration Involving Partial Fractions	To be able to use partial fractions in integration of rational fractions having denominators consisting of: (a) Linear factors (b) Repeated linear factors(up to 3) (c) Linear and non-repeated quadratic

			factors
	6	Equations of Straight lines	(b) To find the slope of a line passing through two points. (f) To transform the linear equation $ax + by + c = 0$ in standard form. (h) To find the area of a triangle whose vertices are given.
	7	Linear Programming	To find the optimal solution of the linear objective functions by graphical methods.
	8	Equation of Hyperbola with given elements	To convert equation of a hyperbola to the standard form by translation of axes and to find the elements.

MATHEMATICS HSSC-II
Table of Specification

Topics	1. Functions and Limits	2.Differentiation	3.Integration	4.Introduction to Analytic Geometry	5.Linear Inequalities and Linear Programming	6.Conic Section	7.Vectors	Total marks of each assessment objectives	% age
Knowledge based	1(1)(1) 2(i)(4) 2(ii)(4)	2(v)(4)	2(vii)(4)	1(10)(1) 1(11)(0.5) 2(x)(4)	1(13)(1) 1(14)(1)	2(xii)(4) 2(xiii)(4)	1(19)(1) 1(20)(1) 2(xvi)(4)	38.5	29.2%
Understanding based	1(2)(1) 1(3)(1) 3(8)	1(4)(1) 1(5)(1) 1(6)(1) 2(iii)(4) 2(iv)(4) 4(8)	1(8)(1) 5(8)	1(11)(0.5) 1(12)(1) 2(ix)(4) 6(8)		1(15)(1) 1(16)(1) 1(17)(1) 2(xiv)(4) 8(4)	1(18)(1) 2(xv)(4)	67.5	51.1%
Application based			1(7)(1) 1(9)(1) 2(vi)(4) 2(viii)(4)		2(xi)(4) 7(8)	8(4)		26	19.7%
Total marks for each topic	19	23	23	19	14	23	11	132	100%

KEY:

1(1)(1)
Question No. (Part No.) (Allocated Marks)