Model Question Paper HSSC-I

Mathematics

(2^{nd} Set)SOLUTION

Section – A

1	D	2	A	3	B	4	D	5	C	6	D	7	C	8	B	9	B	10	A
11	B	12	C	13	B	14	B	15	C	16	A	17	A	18	A	19	C	20	A

SECTION-B

Q2.

(i) If $z = \sqrt{2} - i$ then $\overline{z} = \sqrt{2} + i$

a) $z^2 + \overline{z}^2 = \left(\sqrt{2} - i\right)^2 + \left(\sqrt{2} + i\right)^2$

$z^2 + \overline{z}^2 = 2 - 2\sqrt{2}i + i^2 + 2 + 2\sqrt{2}i + i^2$

$z^2 + \overline{z}^2 = 2 + (-1) + 2 + (-1) = 0 \in \mathcal{R}$

b) $(z - \overline{z})^2 = \left(\sqrt{2} - i - \sqrt{2} - i\right)^2$

$(z - \overline{z})^2 = (-2i)^2$

$(z - \overline{z})^2 = 4i^2 = -4 \in \mathcal{R}$

(ii) $p \longrightarrow q = \sim (p \wedge \sim q)$

p	q	$\sim q$	$\boldsymbol{p \wedge \sim q}$	$\sim (p \wedge \sim q)$	$\boldsymbol{p \longrightarrow q}$
T	T	F	**F**	T	**T**
T	F	T	**T**	F	**F**
F	T	F	**F**	T	**T**
F	F	T	**F**	T	**T**

From column 4 and column 6: $p \longrightarrow q = \sim (p \wedge \sim q)$

(iii) $A = \begin{bmatrix} 1 & 2 & -1 \\ -3 & -2 & 2 \\ 1 & 2 & -3 \end{bmatrix}$

a) $A_{11} = \begin{vmatrix} -2 & 2 \\ 2 & -3 \end{vmatrix} = 6 - 4 = 2$

$A_{21} = \begin{vmatrix} 2 & -1 \\ 1 & -3 \end{vmatrix} - 6 + 1 = -5$

$A_{31} = \begin{vmatrix} 2 & -1 \\ -2 & 2 \end{vmatrix} = 4 + 2 = 6$

b) $|A| = A_{11} - A_{21} + A_{31} = 2 + 5 + 6 = 13$

(iv) $y + 2 = 11x \quad \Rightarrow \quad y = 11x - 2$ ………. eqn-I

$y = 25x^2 - 9x + 2$

Using eqn-I

$11x - 2 = 25x^2 - 9x + 2$

$25x^2 - 20x + 4 = 0$

$(5x - 2)^2 = 0$

$x = \frac{2}{5}$

Substitute value of x in eq-I

$y = 11\left(\frac{2}{5}\right) - 2 = \frac{12}{5}$

Solution Set $= \left\{\left(\frac{2}{5}, \frac{12}{5}\right)\right\}$

(v) $(x - p)(x - q) + (x - q)(x - r) + (x - r)(x - p) = 0$

$x^2 - (p + q)x + pq + x^2 - (q + r)x + qr + x^2 - (r + p)x + rp = 0$

$3x^2 - 2(p + q + r)x + (pq + qr + rp) = 0$

Discriminant $= b^2 - 4ac$

$= [-2(p + q + r)]^2 - 4(3)(pq + qr + rp)$

$= 4(p^2 + q^2 + r^2 + 2pq + 2qr + 2rp - 3pq - 3qr - 3rp)$

$= 4(p^2 + q^2 + r^2 - pq - qr - rp)$

$= 2(2p^2 + 2q^2 + 2r^2 - 2pq - 2qr - 2rp)$

$= 2[p^2 + p^2 + q^2 + q^2 + r^2 + r^2 - 2pq - 2qr - 2rp]$

$= 2[p^2 + q^2 - 2pq + q^2 + r^2 - 2qr + r^2 + +p^2 - 2rp]$

$= 2[(p - q)^2 + (q - r)^2 + (r - p)^2] > 0$

i.e. roots of the given equation are real.

For equal roots

Discriminant $= 0$

$2[(p-q)^2+(q-r)^2+(r-p)^2]=0$

$(p-q)^2+(q-r)^2+(r-p)^2=0$

This is only possible if

$(p-q)^2=0 \quad \Rightarrow \quad p=q$

$(q-r)^2=0 \quad \Rightarrow \quad q=r$

$(r-p)^2=0 \quad \Rightarrow \quad r=p$

Hence $p=q=r$

(vi) $\frac{2x-3}{(x^2-x+1)(3x-2)}=\frac{Ax+B}{(x^2-x+1)}+\frac{C}{(3x-2)}$-----------eqn-I

$2x-3=(Ax+B)(3x-2)+C(x^2-x+1)$---------eqn-II

For C put $3x-2=0$ or $x=\frac{2}{3}$ in eqn-I

$2\left(\frac{2}{3}\right)-3=0+C\left(\frac{4}{9}-\frac{2}{3}+1\right)$

$\frac{4}{3}-3=C\left(-\frac{2}{9}+1\right) \qquad \Rightarrow C=-\frac{15}{7}$

Simplifying eqn-II as

$2x-3=A(3x^2-2x)+B(3x-2)+C(x^2-x+1)$

Equating the coefficients of x^2, constant terms

$0=3A+C \qquad\qquad -3=-2B+C$

$0=3A-\frac{15}{7} \qquad\qquad -3=-2B-\frac{15}{7}$

$A=-\frac{5}{7} \qquad\qquad B=\frac{3}{7}$

$Substituting\ the\ values\ in\ eqn-I$

$$\frac{2x-3}{(x^2-x+1)(3x-2)}=\frac{-\frac{5}{7}x+\frac{3}{7}}{(x^2-x+1)}+\frac{-\frac{15}{7}}{(3x-2)}$$

$$\frac{2x-3}{(x^2-x+1)(3x-2)}=\frac{-5x+3}{7(x^2-x+1)}-\frac{15}{7(3x-2)}$$

(vii) A.P: b,c,p,q,r

Consider the following

$a_1+a_5=b+r \qquad\qquad a_2+a_4=c+q \qquad\qquad 2a_3=2p$

$a+(a+4d)=b+r \qquad\qquad a+d+a+3d=c+q \qquad\qquad 2(a+2d)=2p$

$2a + 4d = b + r$ $\qquad$ $2a + 4d = c + q$ $\qquad$ $2a + 4d = 2p$

From above $\quad b + r = c + q = 2p$

(viii) In HP: $a_p = q \;\; ; \; a_q = p \;\; ; \qquad a_{pq} = ?$

In AP: $a_p = \frac{1}{q} \;\; ; \; a_q = \frac{1}{p}$

$\because a_n = a_1 + (n-1)d$

$a_p = a_1 + (p-1)d$ $\qquad$ $a_q = a_1 + (q-1)d$

$\frac{1}{q} = a_1 + pd - d$ -----eqn-I $\qquad$ $\frac{1}{p} = a_1 + qd - d$

$\frac{1}{q} - \frac{1}{p} = (a_1 + pd - d) - (a_1 + qd - d)d$

$$\frac{1}{q} - \frac{1}{p} = (p-q)d$$

$$\frac{p-q}{pq} = (p-q)d$$

$d = \frac{1}{pq}$

Put the value of d in eqn-I giving $a_1 = \frac{1}{pq}$

$a_{pq} = a_1 + (pq-1)d$

$a_{pq} = \frac{1}{pq} + (pq-1)\frac{1}{pq}$

$$a_{pq} = \frac{1}{pq} + 1 - \frac{1}{pq} = 1$$

$a_{pq} = 1$

Thus $(pq)th$ term of H.P $= 1$

(ix) (a) when C and K are placed together

Number of permutations $= 2! \times 5! = 2 \times 120 = 240$

(b) Total number of permutation= $6! = 720$

when C and K are not placed together

Number of permutations

=Total number of permutation $-$ Number of permutation when C and K are together

$= 720 - 240 = 480$

(x) Let, $a > b$ and $= b + h$, where h is so small that its square and higher power be neglected.

L.H.S $= \frac{b+2a}{a+2b} = \frac{b+2(b+h)}{(b+h)+2b} = \frac{3b+2h}{3b+h} = \frac{1+\frac{2h}{3b}}{1+\frac{h}{3b}}$

$$= \left(1+\frac{2h}{3b}\right)\left(1+\frac{h}{3b}\right)^{-1}$$

$$= \left(1+\frac{2h}{3b}\right)\left(1+(-1)\frac{h}{3b}+\frac{(-1)(-2)}{2!}\left(\frac{h}{3b}\right)^2+\cdots\right)$$

$$\approx \left(1+\frac{2h}{3b}\right)\left(1-\frac{h}{3b}\right)$$

$$\approx 1-\frac{h}{3b}+\frac{2h}{3b}$$

L.H.S $\approx 1+\frac{h}{3b}$

R.H.S. $= \sqrt[3]{\frac{a}{b}} = \left(\frac{b+h}{b}\right)^{\frac{1}{3}} = \left(1+\frac{h}{b}\right)^{\frac{1}{3}} \approx 1+\frac{h}{3b}$

Hence L.H.S = R.H.S

(xi) In the given right triangle where θ lies in the first quadrant

Let $= 12$, $y = 5$, $r = 13$

$sec\theta = \frac{r}{x} = \frac{13}{12}$ $\quad cosec\theta = \frac{r}{y} = \frac{13}{5}$ $\quad tan\theta = \frac{y}{x} = \frac{5}{12}$ $\quad cot\theta = \frac{x}{y} = \frac{12}{5}$

a) $sec^2\theta - tan^2\theta = \left(\frac{13}{12}\right)^2 - \left(\frac{5}{12}\right)^2 = \frac{169}{144} - \frac{25}{144} = \frac{144}{144} = 1$

b) $cosec^2\theta - cot^2\theta = \left(\frac{13}{5}\right)^2 - \left(\frac{12}{5}\right)^2 = \frac{169}{25} - \frac{144}{25} = \frac{25}{25} = 1$

(xii) $\tan(\alpha-\beta) = \frac{tan\alpha - tan\beta}{1+tan\alpha tan\beta}$

$L.H.S = tan\,(\alpha-\beta)$

$$= \frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)}$$

$$= \frac{sin\alpha\, cos\beta - cos\alpha\, sin\beta}{cos\alpha\, cos\beta - sin\alpha\, sin\beta}$$

Dividing the numerator and denominator by $cos\alpha cos\beta$

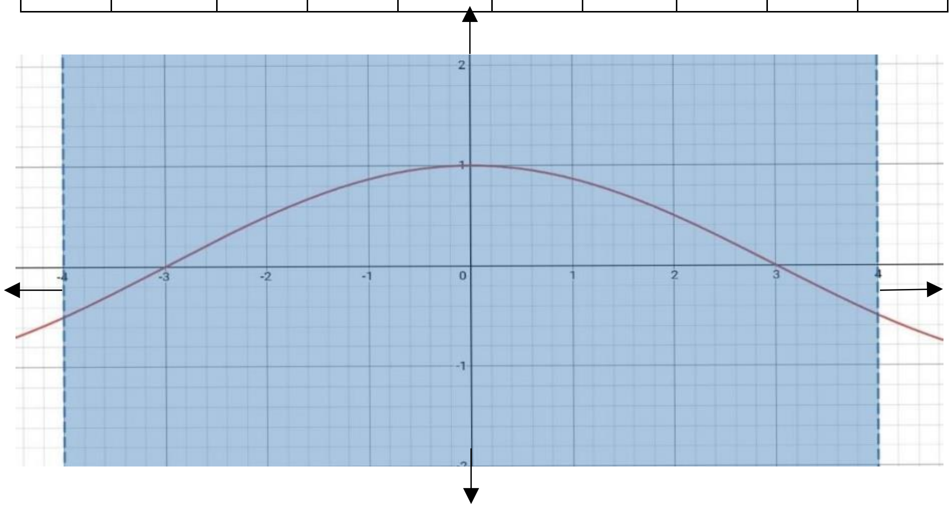
$$= \frac{\left(\frac{sin\alpha\, cos\beta - cos\alpha\, sin\beta}{cos\alpha\, cos\beta}\right)}{\left(\frac{cos\alpha\, cos\beta - sin\alpha\, sin\beta}{\cos\alpha\, cos\beta}\right)}$$

$$= \frac{tan\alpha - tan\beta}{1+tan\alpha tan\beta}$$

= R.H.S.

(xiii) $y = \cos\left(\frac{\pi}{6}\,x\right) \qquad -4 \le x \le 4$

x	-4	-3	-2	-1	0	1	2	3	4
y	-0.5	0	0.5	0.87	1	0.87	0.5	0	0.5



(xiv) $L.H.S = \frac{\cos\alpha}{a} + \frac{\cos\beta}{b} + \frac{\cos\gamma}{c}$

By Law of cosines

$\cos\alpha = \frac{b^2+c^2-a^2}{2bc}$; $\cos\beta = \frac{a^2+c^2-b^2}{2ac}$; $\cos\gamma = \frac{a^2+b^2-c^2}{2ab}$

$L.H.S = \frac{b^2+c^2-a^2}{2abc} + \frac{a^2+c^2-b^2}{2acb} + \frac{a^2+b^2-c^2}{2abc}$

$= \frac{1}{2abc}[b^2 + c^2 - a^2 + a^2 + c^2 - b^2 + a^2 + b^2 - c^2]$

$= \frac{a^2+b^2+c^2}{2abc} =$ R.H.S

(xv) $\text{L.H.S} = 4\tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{239}\right)$

$= 2\left[2\tan^{-1}\left(\frac{1}{5}\right)\right] + \tan^{-1}\left(\frac{1}{239}\right)$

$= 2\left[\tan^{-1}\left(\frac{\frac{2}{5}}{1-\left(\frac{2}{5}\right)^2}\right)\right] + \tan^{-1}\left(\frac{1}{239}\right) \qquad \because 2\tan^{-1}A = \tan^{-1}\left(\frac{2A}{1-A^2}\right)$

$= 2\tan^{-1}\left(\frac{5}{12}\right) + \tan^{-1}\left(\frac{1}{239}\right)$

$= \tan^{-1}\left(\frac{2\times\frac{5}{12}}{1-\left(\frac{5}{12}\right)^2}\right) + \tan^{-1}\left(\frac{1}{239}\right)$

$= \tan^{-1}\left(\frac{120}{119}\right) + \tan^{-1}\left(\frac{1}{239}\right)$

$= \tan^{-1}\left(\frac{\frac{120}{119}+\frac{1}{239}}{1-\left(\frac{120}{119}\times\frac{1}{239}\right)}\right)$ $\because \tan^{-1} A + \tan^{-1} B = \tan^{-1}\left(\frac{A+B}{1-AB}\right)$

$= \tan^{-1}\left(\frac{28680-119}{28441+120}\right) = \tan^{-1}\left(\frac{28561}{28561}\right)$

$= \tan^{-1}(1)$

$= \frac{\pi}{4} = R.H.S$

(xvi) $sinx + cosx = 1$

Taking square of both sides

$\sin^2 x + \cos^2 x + 2sinxcosx = 1$

$1+2sinxcosx = 1$

$2sinxcosx = 0$

$\sin 2x = 0$

$2x = n\pi$ or $x = \frac{n\pi}{2}$ where $n \in$ Z

Solution set= $\left\{\frac{n\pi}{2}\right\}$, $n \in$ Z

SECTION-C

Q3. $x + 7y - z = 10$

$4x + 8y + z = 5$

$2x - 3y + 2z = -5$

$$\begin{bmatrix} 1 & 7 & -1 & 10 \\ 4 & 8 & 1 & 5 \\ 2 & -3 & 2 & -5 \end{bmatrix}$$

$$R\sim\begin{bmatrix} 1 & 7 & -1 & 10 \\ 0 & -20 & 5 & -35 \\ 0 & -17 & 4 & -25 \end{bmatrix} \quad R_2 - 4R_1 \; ; \; R_3 - 2R_1$$

$$R\sim\begin{bmatrix} 1 & 7 & -1 & 10 \\ 0 & 1 & -\frac{1}{4} & \frac{7}{4} \\ 0 & -17 & 4 & -25 \end{bmatrix} \quad -\frac{1}{20}R_2$$

$$R \sim \begin{bmatrix} 1 & 0 & \frac{3}{4} & -\frac{9}{4} \\ 0 & 1 & -\frac{1}{4} & \frac{7}{4} \\ 0 & 0 & -\frac{1}{4} & \frac{19}{4} \end{bmatrix} \qquad R_1 - 7R_2 \; ; \; R_3 + 17R_2$$

$$R \sim \begin{bmatrix} 1 & 0 & \frac{3}{4} & -\frac{9}{4} \\ 0 & 1 & -\frac{1}{4} & \frac{7}{4} \\ 0 & 0 & 1 & -19 \end{bmatrix} \qquad -4R_3$$

$$R \sim \begin{bmatrix} 1 & 0 & 0 & 12 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & -19 \end{bmatrix} \qquad \frac{1}{4}R_3 + R_2 \; ; \; -\frac{3}{4}R_3 + R_1$$

$x = 12,\ y = -3,\ z = -19$

Q4. $ax^2 + bx + c = 0\ ,\ (a \neq 0)$

(i) Let α and 3α be the roots of the equation

Sum of the roots: $\alpha + 3\alpha = -\frac{b}{a}$

$$4\alpha = -\frac{b}{a} \qquad \Longrightarrow \alpha = -\frac{b}{4a}$$

Product of the roots: $(\alpha)(3\alpha) = \frac{c}{a}$

$$3\alpha^2 = \frac{c}{a}$$

$$3\left(-\frac{b}{4a}\right)^2 = \frac{c}{a} \qquad \because \alpha = -\frac{b}{4a}$$

$$\Longrightarrow 3b^2 = 16ca$$

(ii) Let α and α^2 be the roots of the equation

Product of the roots: $(\alpha)(\alpha^2) = \frac{c}{a}$

$$\alpha^3 = \frac{c}{a} \qquad \Longrightarrow \quad \alpha = \left(\frac{c}{a}\right)^{\frac{1}{3}}$$

Sum of the roots: $\alpha + \alpha^2 = -\frac{b}{a}$

$$\left(\frac{c}{a}\right)^{\frac{1}{3}} + \left(\frac{c}{a}\right)^{\frac{2}{3}} = -\frac{b}{a} \qquad \because \alpha = \left(\frac{c}{a}\right)^{\frac{1}{3}}$$

(iii) Let α and $-\alpha$ be the roots of the equation, then

Product of the roots: $(\alpha)(-\alpha) = \frac{c}{a}$

$$\alpha^2 = -\frac{c}{a}$$

Sum of the roots: $\alpha + (-\alpha) = -\frac{b}{a}$

$$0 = -\frac{b}{a}$$

$$b = 0$$

(iv) Let α and $\frac{1}{\alpha}$ be the roots of the equation.

Sum of the roots: $\alpha + \frac{1}{\alpha} = -\frac{b}{a}$

Product of the roots: $(\alpha)\left(\frac{1}{\alpha}\right) = \frac{c}{a}$

$$1 = \frac{c}{a}$$

$$c = a$$

Q5. Let $x = \left(2^{\frac{1}{4}}\right)\left(4^{\frac{1}{8}}\right)\left(8^{\frac{1}{16}}\right)\left(16^{\frac{1}{32}}\right) \ldots \infty$

Taking log on both sides

$$logx = log2^{\frac{1}{4}} + log4^{\frac{1}{8}} + log8^{\frac{1}{16}} + log16^{\frac{1}{32}} + \cdots \infty$$

$$logx = log2^{\frac{1}{4}} + log(2^2)^{\frac{1}{8}} + log(2^3)^{\frac{1}{16}} + log(2^4)^{\frac{1}{32}} + \cdots \infty$$

$$logx = \frac{1}{4}log2 + \frac{2}{8}log2 + \frac{3}{16}log2 + \frac{4}{32}log2 + \cdots \infty$$

$$logx = \left(\frac{1}{4} + \frac{2}{8} + \frac{3}{16} + \frac{4}{32} + \cdots \infty\right) log2 \quad \ldots\ldots\ldots\ldots \quad \text{(i)}$$

Let $S = \frac{1}{4} + \frac{2}{8} + \frac{3}{16} + \frac{4}{32} + \cdots \infty$ ……….. (ii)

Multiplying by $\left(\frac{1}{2}\right)$

$$\frac{1}{2}S = \frac{1}{8} + \frac{2}{16} + \frac{3}{32} + \frac{4}{64} + \cdots \infty \quad \ldots\ldots\ldots\ldots \quad \text{(iii)}$$

eq. (ii) – eq. (iii)

$$\frac{1}{2}S = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots \infty$$

$$\frac{1}{2}S = S_\infty$$

Where $S_\infty = \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \frac{1}{2^5} + \cdots \infty$

Here $a_1 = \frac{1}{2^2}$ and $r = \frac{1}{2}$

$\therefore \quad S_\infty = \frac{a_1}{1-r} = \frac{\frac{1}{2^2}}{\frac{1}{2}} = \frac{1}{2}$

Now $\frac{1}{2}S = \frac{1}{2}$

$S = 1$

Put it in eq. (i)

$logx = (S)log2$

$logx = (1)log2$

$logx = log2$

$x = 2$

Q6. $3^n + 2^{n-1} < 4^n$

Step 1: Let the given inequality is true for $n = 2$

$3^2 + 2^1 < 4^2$

$9 + 2 < 16$

$11 < 16$ (true)

Step 2: Let the statement be true for $n = k \geq 2$

$3^k + 2^{k-1} < 4^k$ ……….. (i)

Step 3: Multiplying both sides by 4

$4(3^k + 2^{k-1}) < 4(4^k)$

$4 \cdot 3^k + 4 \cdot 2^{k-1} < 4 \cdot 4^k$

$(3 + 1) \cdot 3^k + (2 + 2) \cdot 2^{k-1} < 4^{k+1}$

$3 \cdot 3^k + 3^k + 2 \cdot 2^{k-1} + 2 \cdot 2^{k-1} < 4^{k+1}$

$3^{k+1} + 2^k + 3^k + 2^k < 4^{k+1}$

$3^{\overline{k+1}} + 2^{\overline{k+1}-1} < 4^{\overline{k+1}}$

Which is true for $n = k + 1$.

The truth for $n = k$ implies its truth for $n = k + 1$.

Hence it is true for all positive integral values of n.

Q7.

(i) $sin3\theta + sin5\theta + sin7\theta + sin9\theta = 4cos\theta sin6\theta cos2\theta$

L.H.S. = $sin3\theta + sin5\theta + sin7\theta + sin9\theta$

Using $sinA + sinB = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

L.H.S. = $2\sin\left(\frac{3\theta+5\theta}{2}\right)\cos\left(\frac{3\theta-5\theta}{2}\right) + 2\sin\left(\frac{7\theta+9\theta}{2}\right)\cos\left(\frac{7\theta-9\theta}{2}\right)$

L.H.S. = $2\sin 4\theta\cos\theta + 2\sin 8\theta\cos\theta$

L.H.S. = $2cos\theta(sin4\theta + sin8\theta)$

L.H.S. $= 2cos\theta\left\{2\sin\left(\frac{4\theta+8\theta}{2}\right)\cos\left(\frac{4\theta-8\theta}{2}\right)\right\}$

L.H.S. $= 4cos\theta sin6\theta cos2\theta$ =R.H.S.

(ii) $cos5\theta + cos\theta + 2cos3\theta = 4cos3\theta\cos^2\theta$

L.H.S. $= cos5\theta + cos\theta + 2cos3\theta$

Using $cosA + cosB = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

L.H.S. $= 2\cos\left(\frac{5\theta+\theta}{2}\right)\cos\left(\frac{5\theta-\theta}{2}\right) + 2cos3\theta$

L.H.S. $= 2\cos 3\theta\cos 2\theta + 2\cos 3\theta$

L.H.S. $= 2cos3\theta(cos2\theta + 1)$

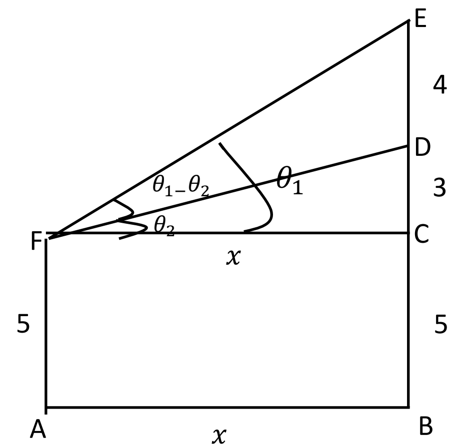
L.H.S. $= 2cos3\theta(2\cos^2\theta)$ $\quad\because cos2\theta + 1 = 2\cos^2\theta$

L.H.S. $= 4cos3\theta\cos^2\theta$ =R.H.S.

Q8. Let height of the poster: $|DE| = 4feet$

Height of the man: $|AF| = 5\ feet$

Angle of Observation: $\angle D\hat{E}F = \theta = \theta_1 - \theta_2 = ?$



In rt$\triangle$ EFC $\tan\theta_1 = \frac{7}{x}$

In rt$\triangle$ DFC $\tan\theta_2 = \frac{3}{x}$

$$\tan(\theta_1 - \theta_2) = \frac{\tan\theta_1 - \tan\theta_2}{1 + \tan\theta_1\tan\theta_2}$$

$$\tan(\theta) = \frac{\frac{7}{x} - \frac{3}{x}}{1 + \left(\frac{7}{x}\right)\left(\frac{3}{x}\right)} = \frac{\frac{4}{x}}{\frac{x^2+21}{x^2}} = \frac{4x}{x^2+21}$$

$$\theta = \tan^{-1}\left(\frac{4x}{x^2+21}\right)$$