



Model Question Paper HSSC – II (3rd Set Solution)

Mathematics

SECTION – A (Marks 20)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
A	D	B	B	D	C	C	A	A	D	C	B	B	C	D	B	C	B	D	D

SECTION – B

(i) $u(x) = \sqrt{x^2 - 3}$ and $v(x) = x^2 + 3$

(a) $u(x) = \sqrt{x^2 - 3}$

Substituting $x = v(x)$

$$\Rightarrow u(v(x)) = \sqrt{(v(x))^2 - 3} \quad [01 \text{ Mark}]$$

$$\Rightarrow u(x^2 + 3) = \sqrt{(x^2 + 3)^2 - 3} = \sqrt{x^4 + 6x^2 + 6} \quad [01 \text{ Mark}]$$

(b) $v(x) = x^2 + 3$

Substituting $x = u(x)$

$$\Rightarrow v(u(x)) = (u(x))^2 + 3 \quad [01 \text{ Mark}]$$

$$\Rightarrow v(\sqrt{x^2 - 3}) = (\sqrt{x^2 - 3})^2 + 3 = x^2 \quad [01 \text{ Mark}]$$

(ii) Let $f(x) = \begin{cases} \frac{\sin 5x}{x} & \text{if } x < 0 \\ (1-x)^{-5/x} & \text{if } x \geq 0 \end{cases}$,

(a) $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \frac{\sin 5x}{x}$

Multiplying and Dividing by 5

$$\Rightarrow \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \frac{\sin 5x}{x} \times \frac{5}{5} = 5 \lim_{x \rightarrow 0^-} \frac{\sin 5x}{5x} \quad [01 \text{ Mark}]$$

Applying limit by using $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

$$\Rightarrow \lim_{x \rightarrow 0^-} f(x) = 5(1) = 5 \quad [01 \text{ Mark}]$$

(b) $\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} (1-x)^{-5/x}$

$$\Rightarrow \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} [1 + (-x)]^{\frac{1}{(-x)} \times 5} \quad [01 \text{ Mark}]$$

Applying limit by using $\lim_{x \rightarrow 0} (1 + x)^{1/x} = e$

$$\Rightarrow \lim_{x \rightarrow 0^+} f(x) = e^5 \quad [01 \text{ Mark}]$$

(iii) Let $y = u(x)v(x)$

Taking increment

$$y + \delta y = u(x + \delta x)v(x + \delta x) \quad [01 \text{ Mark}]$$

$$\Rightarrow \delta y = u(x + \delta x)v(x + \delta x) - u(x)v(x)$$

Adding and Subtracting $u(x + \delta x)v(x)$ on R.H.S.

$$\Rightarrow \delta y = u(x + \delta x)v(x + \delta x) - u(x + \delta x)v(x) + u(x + \delta x)v(x) - u(x)v(x) \quad [01 \text{ Mark}]$$

$$\Rightarrow \delta y = u(x + \delta x)(v(x + \delta x) - v(x)) + v(x)(u(x + \delta x) - u(x))$$

Dividing δx on bothsides

$$\Rightarrow \frac{\delta y}{\delta x} = u(x + \delta x) \left(\frac{v(x + \delta x) - v(x)}{\delta x} \right) + v(x) \left(\frac{u(x + \delta x) - u(x)}{\delta x} \right) \quad [01 \text{ Mark}]$$

Taking $\lim_{\delta x \rightarrow 0}$ on bothsides

$$\Rightarrow \lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = \lim_{\delta x \rightarrow 0} (u(x + \delta x)) \lim_{\delta x \rightarrow 0} \left(\frac{v(x + \delta x) - v(x)}{\delta x} \right) + v(x) \lim_{\delta x \rightarrow 0} \left(\frac{u(x + \delta x) - u(x)}{\delta x} \right)$$

Applying limit

$$\Rightarrow \frac{dy}{dx} = u(x)v'(x) + v(x)u'(x) \quad [01 \text{ Mark}]$$

(iv) $y = \frac{1 + \csc x}{1 - \csc x}$

Differentiating y w.r.t. x .

$$\Rightarrow \frac{d}{dx} (y) = \frac{d}{dx} \left(\frac{1 + \csc x}{1 - \csc x} \right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{(1 - \csc x) \frac{d}{dx} (1 + \csc x) - (1 + \csc x) \frac{d}{dx} (1 - \csc x)}{(1 - \csc x)^2} \quad [02 \text{ Marks}]$$

$$\Rightarrow \frac{dy}{dx} = \frac{(1 - \csc x)(0 - \csc x \cot x) - (1 + \csc x)(0 + \csc x \cot x)}{(1 - \csc x)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\csc x \cot x (-1 + \csc x - 1 - \csc x)}{(1 - \csc x)^2} = \frac{-2 \csc x \cot x}{(1 - \csc x)^2} \quad [02 \text{ Marks}]$$

(v) $x^2 = \frac{x+y}{x-y}$

$$\Rightarrow x^2(x - y) = x + y$$

$$\Rightarrow x^3 - x^2 y = x + y$$

Differentiating w.r.t. x

$$\Rightarrow \frac{d}{dx}(x^3) - \frac{d}{dx}(x^2y) = \frac{d}{dx}(x) + \frac{d}{dx}(y)$$

$$\Rightarrow 3x^2 - x^2 \frac{dy}{dx} - 2xy = 1 + \frac{dy}{dx}$$

$$\Rightarrow (1 + x^2) \frac{dy}{dx} = 3x^2 - 2xy - 1$$

[02 Marks]

Again, differentiating w.r.t. x

$$\Rightarrow (1 + x^2) \frac{d^2y}{dx^2} + 2x \frac{dy}{dx} = 6x - 2x \frac{dy}{dx} - 2y$$

$$\Rightarrow (1 + x^2) \frac{d^2y}{dx^2} = 6x - 2y - 4x \frac{dy}{dx}$$

$$\Rightarrow (1 + x^2) \frac{d^2y}{dx^2} = 6x - 2y - 4x \left(\frac{3x^2 - 2xy - 1}{1 + x^2} \right)$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{6x + 6x^3 - 2y - 2yx^2 - 12x^3 + 8x^2y + 4x}{1 + x^2}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{10x - 6x^3 - 2y + 6yx^2}{1 + x^2}$$

[02 Marks]

(vi) Let $I = \int \frac{x}{\sqrt{x-1}} dx$

Substituting $x - 1 = t^2 \Rightarrow x = t^2 + 1$

$$\Rightarrow dx = 2t dt$$

[02 Marks]

$$I = \int \frac{t^2+1}{t} (2t dt)$$

$$I = 2 \int (t^2 + 1) dt \Rightarrow I = \frac{2t^3}{3} + 2t + C$$

[02 Marks]

(vii) $(1 + x^2)y_1 = 1$ with $y(1) = 0$

$$\Rightarrow (1 + x^2) \frac{dy}{dx} = 1$$

By Separation of Variables

$$\Rightarrow dy = \frac{1}{1+x^2} dx$$

[01 Mark]

Integrating on bothsides

$$\Rightarrow \int dy = \int \frac{1}{1+x^2} dx$$

$$\Rightarrow y = \tan^{-1} x + C \rightarrow (i)$$

[01 Mark]

Using the initial condition; $y(x = 1) = 0$

$$\Rightarrow 0 = \tan^{-1}(1) + C \Rightarrow C = -\frac{\pi}{4}$$

[01 Mark]

Substituting $C = -\frac{\pi}{4}$ in Eq. (i)

$$\Rightarrow y = \tan^{-1} x - \frac{\pi}{4}$$

[01 Mark]

(viii) $\int_{-1}^1 (1 - |x|) dx$

$$\begin{aligned}
&= \int_{-1}^0 (1 - (-x))dx + \int_0^1 (1 - (+x))dx \\
&= \int_{-1}^0 (1 + x)dx + \int_0^1 (1 - x)dx && \text{[02 Marks]} \\
&= \left[x + \frac{x^2}{2} \right]_{-1}^0 + \left[x - \frac{x^2}{2} \right]_0^1 \\
&= \left[(0 + 1) + \frac{1}{2}(0 - 1) \right] + \left[(1 - 0) - \frac{1}{2}(1 - 0) \right] \\
&= \left(1 - \frac{1}{2} \right) + \left(1 - \frac{1}{2} \right) = \frac{1}{2} + \frac{1}{2} = 1 && \text{[02 Marks]}
\end{aligned}$$

(ix) Let $R(x, y)$ be the required point. [01 Mark]
Since, $|PR| = |QR|$ for $P(3,7)$ and $Q(11, -6)$

$$\begin{aligned}
&\Rightarrow \sqrt{(x - 3)^2 + (y - 7)^2} = \sqrt{(x - 11)^2 + (y + 6)^2} \\
&\Rightarrow (x - 3)^2 + (y - 7)^2 = (x - 11)^2 + (y + 6)^2 && \text{[01 Mark]} \\
&\Rightarrow x^2 - 6x + 9 + y^2 - 14y + 49 = x^2 - 22x + 121 + y^2 + 12y + 36 \\
&\Rightarrow -6x + 58 - 14y = -22x + 157 + 12y \\
&\Rightarrow 16x - 26y = 99 \quad \text{---} \rightarrow (i) && \text{[01 Mark]}
\end{aligned}$$

Since the point R is on the y -axis, so $x = 0$
 \therefore Eq. (i) $\Rightarrow 16(0) - 26y = 99 \Rightarrow y = -\frac{99}{26} = -3.8$
Hence, $R(0, -3.8)$ is the required point. [01 Mark]

(x) $2x - 5 = ky \quad \text{---} \rightarrow (i)$
 $(k + 1)x = 6y - 3 \quad \text{---} \rightarrow (ii)$
Eq. (i) $\Rightarrow y = \frac{2}{k}x - \frac{5}{k} \Rightarrow m_1 = \frac{2}{k}$ [01 Mark]
Eq. (ii) $\Rightarrow y = \frac{k+1}{6}x + \frac{1}{2} \Rightarrow m_2 = \frac{k+1}{6}$ [01 Mark]

Since, the two lines have the same gradients.
 $\Rightarrow m_1 = m_2$
 $\Rightarrow \frac{2}{k} = \frac{k+1}{6}$ [01 Mark]
 $\Rightarrow k^2 + k - 12 = 0$
 $\Rightarrow k = -4$ and $k = 3$ [01 Mark]

(xi) First we find the equation of the line joining $A(0,6)$ and $B(8,0)$. [01 Mark]
 $\Rightarrow \frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$
 $\Rightarrow \frac{y - 6}{0 - 6} = \frac{x - 0}{8 - 0} \Rightarrow 3x + 4y = 24 \quad \text{---} \rightarrow (i)$ [01 Mark]

Another line is; $y = x + 1 \quad \text{---} \rightarrow (ii)$
Substituting the value of y from Eq. (ii) in Eq. (i), we obtain
 $\Rightarrow 3x + 4(x + 1) = 24 \Rightarrow 7x = 20$
 $\Rightarrow x = \frac{20}{7}$, Putting the value of x in Eq. (ii)
 $\Rightarrow y = \frac{20}{7} + 1 = \frac{27}{7}$

Hence, the coordinates of point M , intersecting Eq. (i) and Eq. (ii) are $x = \frac{20}{7}$ and $y = \frac{27}{7}$ [02 Marks]

(xii) Focus: $(3,0)$ and the equation of the directrix is; $x - 1 = 0$.
Where, $F(c, 0) = (3,0) \Rightarrow c = 3$

And, Directrix is $x = \frac{a}{e} = 1 \Rightarrow a = e$ [01 Mark]

$\Rightarrow e = \frac{c}{a} = \frac{3}{e} \Rightarrow e^2 = 3 \Rightarrow e = \sqrt{3}$ [01 Mark]

By the definition of hyperbola

$|PF| = e \cdot |PD|$

$\Rightarrow \sqrt{(x-3)^2 + (y-0)^2} = \sqrt{3}|x-1|$

$\Rightarrow x^2 - 6x + 9 + y^2 = 3(x^2 - 2x + 1)$

$\Rightarrow x^2 - 6x + y^2 = 3x^2 - 6x + 3 - 9$

$\Rightarrow -2x^2 + y^2 = -6$

$\Rightarrow \frac{x^2}{3} - \frac{y^2}{6} = 1$ [02 Marks]

(xiii) Eccentricity; $e = 0.8$, and

Foci; $F(0, \pm c) = (0, \pm 8)$

Since, $e = \frac{c}{a} = 0.8$

$\Rightarrow \frac{8}{a} = 0.8, \Rightarrow a = 10$

Or the vertices are $(0, \pm 10)$ [02 Marks]

Now, Equation of the ellipse by definition is;

$|PF| + |PF'| = 2a$

$\Rightarrow \sqrt{(x-0)^2 + (y-8)^2} + \sqrt{(x-0)^2 + (y+8)^2} = 2(10)$

$\Rightarrow \sqrt{x^2 + y^2 - 16y + 64} = 20 - \sqrt{x^2 + y^2 + 16y + 64}$

$\Rightarrow x^2 + y^2 - 16y + 64 = 400 + x^2 + y^2 + 16y + 64 - 40\sqrt{x^2 + y^2 + 16y + 64}$

$\Rightarrow 5\sqrt{x^2 + y^2 + 16y + 64} = 4y + 50$

$\Rightarrow 25x^2 + 25y^2 + 80y + 320 = 16y^2 + 400y + 2500$

$\Rightarrow 25x^2 + 9y^2 - 320y - 2180 = 0$ [02 Marks]

(xiv) $\underline{u} = 3\hat{i} - 2\hat{j} + 6\hat{k}$

Length of \underline{u} : $|\underline{u}| = \sqrt{3^2 + (-2)^2 + 6^2} = 7$ [01 Mark]

Direction of \underline{u} : $\frac{\underline{u}}{|\underline{u}|} = \frac{3\hat{i} - 2\hat{j} + 6\hat{k}}{7} = \frac{3}{7}\hat{i} - \frac{2}{7}\hat{j} + \frac{6}{7}\hat{k}$ [01 Mark]

Product of the length and direction of \underline{u} is:

$\Rightarrow |\underline{u}| \left(\frac{\underline{u}}{|\underline{u}|} \right) = 7 \left(\frac{3}{7}\hat{i} - \frac{2}{7}\hat{j} + \frac{6}{7}\hat{k} \right)$

$\Rightarrow |\underline{u}| \left(\frac{\underline{u}}{|\underline{u}|} \right) = 3\hat{i} - 2\hat{j} + 6\hat{k}$ [02 Marks]

(xv) Force: $F = \hat{i} - 2\hat{j} + 3\hat{k}$

And origin $O(0,0,0)$ to the point $A(2, -1, 4)$

Position vector of \overrightarrow{OA} is:

$\vec{d} = \overrightarrow{OA} = (2-0)\hat{i} + (-1-0)\hat{j} + (4-0)\hat{k}$

$\Rightarrow \vec{d} = \overrightarrow{OA} = 2\hat{i} - \hat{j} + 4\hat{k}$ [01 Mark]

Now, Work done = $\vec{F} \cdot \vec{d}$ [01 Mark]

$\Rightarrow W = (\hat{i} - 2\hat{j} + 3\hat{k}) \cdot (2\hat{i} - \hat{j} + 4\hat{k})$

$\Rightarrow W = 1(2) + (-2)(-1) + 3(4) = 16 \text{ Joule}$ [02 Marks]

(xvi) The vertices of a triangle are $A(1, -1, 0)$, $B(2, 1, -1)$ and $C(-1, 1, 2)$.

Let $\underline{a} = \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (1, 2, -1)$

And $\underline{b} = \overrightarrow{CA} = \overrightarrow{OA} - \overrightarrow{OC} = (2, -2, -2)$

[01 Mark]

Now, Area of triangle $= \frac{1}{2} |\underline{a} \times \underline{b}|$

$$\underline{a} \times \underline{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 2 & -2 & -2 \end{vmatrix}$$

Expanding for first Row, we obtain

$$= -6\hat{i} - 0\hat{j} - 6\hat{k}$$

[02 Marks]

$$\text{Area of triangle} = \frac{1}{2} |\underline{a} \times \underline{b}| = \frac{1}{2} \sqrt{(-6)^2 + 0^2 + (-6)^2}$$

$$= \frac{1}{2} \sqrt{36 + 36} = \frac{6}{2} \sqrt{2} = 3\sqrt{2} \text{ sq. units}$$

[01 Mark]

SECTION – C

Q.3 Solution:

$$f(x) = \begin{cases} x^2 - 1 & \text{if } 2 < x < 3 \\ 2ax & \text{if } x = 3 \\ bx - 4 & \text{if } x > 3 \end{cases} \text{ is continuous } \forall x.$$

First, we find the value of $f(x)$ at $x = 3$

(i.e.) $f(x) = 2ax$

Substituting $x = 3$, we get

$$\Rightarrow f(3) = 2a(3) = 6a \quad \text{--- --> (i)}$$

[02 Marks]

Second, we find limit of $f(x)$ at $x = 3$

L.H.Limit:

$$\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} (x^2 - 1)$$

Applying limit on R.H.S.

$$\Rightarrow \lim_{x \rightarrow 3^-} f(x) = 3^2 - 1 = 8 \quad \text{--- --> (ii)}$$

R.H.Limit:

$$\lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} (bx - 4)$$

Applying limit on R.H.S.

$$\Rightarrow \lim_{x \rightarrow 3^+} f(x) = b(3) - 4 = 3b - 4 \quad \text{--- --> (iii)}$$

[02 Marks]

For continuous function, $\lim_{x \rightarrow 3} f(x)$ exists, so

$$\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^+} f(x)$$

From Eq. (ii) and (iii), we obtain

$$8 = 3b - 4$$

$$\Rightarrow b = 4$$

[02 Marks]

Third, $f(3) = \lim_{x \rightarrow 3} f(x)$

$$\Rightarrow 6a = 8$$

$$\Rightarrow a = 4/3$$

[02 Marks]

Q.4 Solution:

We have,

$$y = ae^x + be^{2x} + ce^{3x}, \text{ where } a, b, c \in R$$

We shall show that

$$y_3 - 6y_2 + 11y_1 - 6y = 0 \quad \text{--- --> (i)}$$

Differentiating w.r.t. (x)

$$\frac{d}{dx}(y) = a \frac{d}{dx}(e^x) + b \frac{d}{dx}(e^{2x}) + c \frac{d}{dx}(e^{3x})$$

$$\Rightarrow y_1 = ae^x + 2b(e^{2x}) + 3c(e^{3x})$$

$$\Rightarrow y_1 = ae^x + 2be^{2x} + 3ce^{3x} \quad [02 \text{ Marks}]$$

Again, diff. w.r.t. (x)

$$y_2 = ae^x + 2be^{2x}(2) + 3ce^{3x}(3)$$

$$\Rightarrow y_2 = ae^x + 4be^{2x} + 9ce^{3x} \quad [02 \text{ Marks}]$$

Again, diff. w.r.t. (x)

$$y_3 = ae^x + 4be^{2x}(2) + 9ce^{3x}(3)$$

$$y_3 = ae^x + 8be^{2x} + 27ce^{3x} \quad [02 \text{ Marks}]$$

Taking L.H.S. of Eq. (i)

$$L.H.S. = y_3 - 6y_2 + 11y_1 - 6y$$

$$= (ae^x + 8be^{2x} + 27ce^{3x}) - 6(ae^x + 4be^{2x} + 9ce^{3x}) + 11(ae^x + 2be^{2x} + 3ce^{3x})$$

$$- 6(ae^x + be^{2x} + ce^{3x})$$

$$= ae^x + 8be^{2x} + 27ce^{3x} - 6ae^x - 24be^{2x} - 54ce^{3x} + 11ae^x + 22be^{2x} + 33ce^{3x} - 6ae^x$$

$$- 6be^{2x} - 6ce^{3x}$$

$$= 0 = R.H.S. \text{ (Proved)} \quad [02 \text{ Marks}]$$

Q.5 Solution:

$$\int_1^3 \frac{\ln x \sin(\ln x)}{x} dx$$

Substituting $\ln x = t$

$$\Rightarrow \frac{1}{x} dx = dt$$

As $x \rightarrow 1$, then $t \rightarrow 0$

As $x \rightarrow 3$, then $t \rightarrow \ln 3$

[04 Marks]

$$\int_1^3 \frac{\ln x \sin(\ln x)}{x} dx = \int_0^{\ln 3} t \sin t dt$$

$$= \left[t \int \sin t dt - \int \left\{ \frac{d}{dt}(t) \int \sin t dt \right\} dt \right]_0^{\ln 3}$$

$$= [-t \cos t]_0^{\ln 3} - \int_0^{\ln 3} (-\cos t) dt$$

$$= -(\ln 3 \cos(\ln 3) - 0) + [\sin t]_0^{\ln 3}$$

$$= -\ln 3 \cos(\ln 3) + \sin(\ln 3)$$

$$= \sin(\ln 3) - \ln 3 \cos(\ln 3)$$

[04 Marks]

Q.6 Solution:

a) Let $P(x, y)$ be the required point.

Since, $\Delta PAB = 8 \text{ sq. units}$

$$\text{Where } \Delta PAB = \frac{1}{2} \begin{vmatrix} 2 & 3 & 1 \\ 3 & 5 & 1 \\ x & y & 1 \end{vmatrix} = 8$$

Expanding for R_3

$$\Rightarrow x \begin{vmatrix} 3 & 1 \\ 5 & 1 \end{vmatrix} - y \begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} + 1 \begin{vmatrix} 2 & 3 \\ 3 & 5 \end{vmatrix} = 16$$

$$\Rightarrow x(-2) - y(-1) + 1(1) = 16$$

$$\Rightarrow -2x - y = 15$$

$$\Rightarrow 2x + y = -15$$

----- (i)

[04 Marks]

b) Since $P(x, y)$ is equidistant from $A(2, 3)$ and $B(3, 5)$, then

$$|PA|^2 = |PB|^2$$

$$\Rightarrow (x - 2)^2 + (y - 3)^2 = (x - 3)^2 + (y - 5)^2$$

$$\Rightarrow x^2 - 4x + 4 + y^2 - 6y + 9 = x^2 - 6x + 9 + y^2 - 10y + 25$$

$$\Rightarrow 2x + 4y = 21 \quad \text{---} \rightarrow (ii)$$

Solving Eq. (i) and (ii) simultaneously, we get

$$\Rightarrow x = -27/2 \text{ and } y = 12$$

[04 Marks]

Q.7 Solution:

a) Let the mixture contains x kg of food A and y kg of Food B.

Cost of food A is 50 and cost of food B is 70 per kg.

$$\text{Total cost is: } z = 50x + 70y$$

For vitamin A:

Food A contains 3 units and food B contains 2 units and total unit is at-most 9.

$$\Rightarrow 3x + 2y \leq 9 \quad \text{---} \rightarrow (i)$$

For Vitamin C:

Food A contains 2 units and food B contains 3 units and total unit is at-most 12.

$$\Rightarrow 2x + 3y \leq 12 \quad \text{---} \rightarrow (ii)$$

Subject to the constraints $x \geq 0, y \geq 0$

The associated equations of (i) and (ii) are:

$$3x + 2y = 9$$

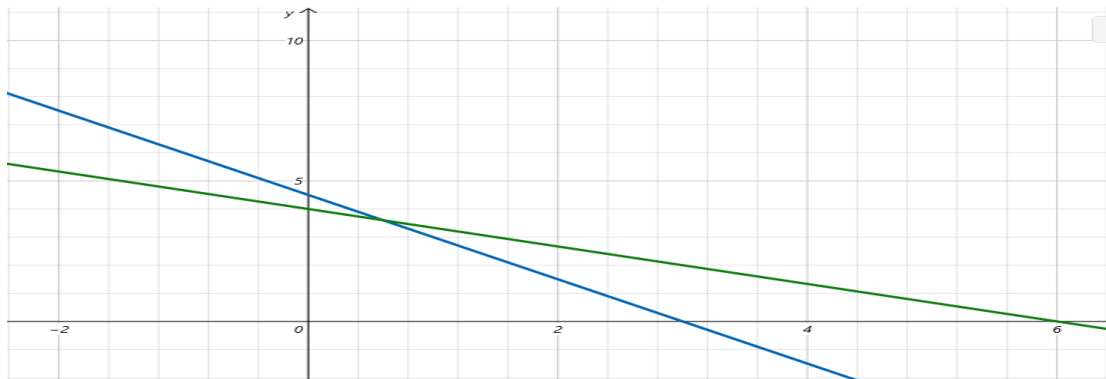
The points satisfying the associated equation are (3,0) and (0,4.5).

$$2x + 3y = 12$$

The points satisfying the associated equation are (6,0) and (0,4).

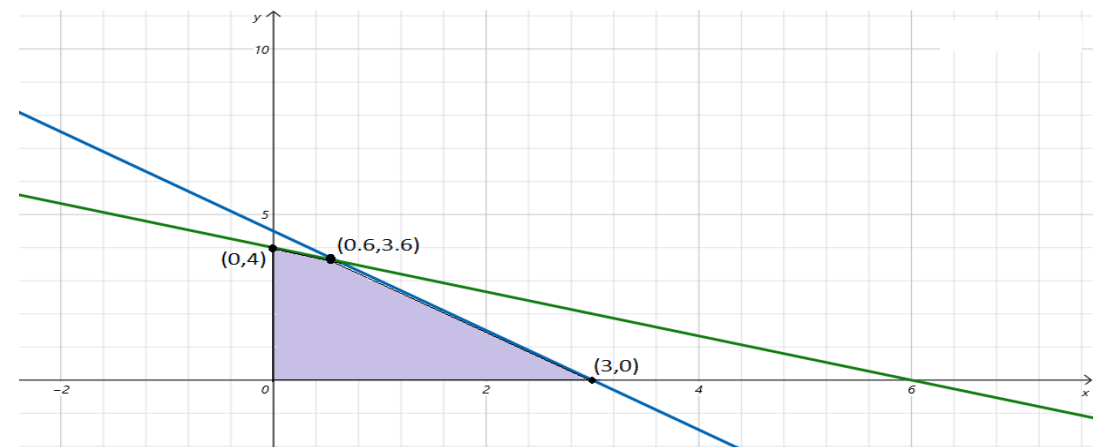
[02 Marks]

b)



[02 Marks]

c)



[02 Marks]

d) The corner points are (0,4), (0.6,3.6) and (3,0).

$$\text{At } (0,4) \Rightarrow z = 50(0) + 70(4) = 280$$

At (0.6,3.6) $\Rightarrow z = 50(0.6) + 70(3.6) = 282$

At (3,0) $\Rightarrow z = 50(3) + 70(0) = 150$

Hence, the optimal solution is $x = 0.6$ and $y = 3.6$ and minimum $z = 150$.

Thus, the minimum cost is Rs. 150 and the mixture has 0.6kg of Food A and 3.6kg of Food B.

[02 Marks]

Q.8 Solution:

Equation of circle is: $x^2 + y^2 - 2x - 10y + 1 = 0$ --- (i)

And equation of the line intersecting the circle (i) is: $x - 2y + 7 = 0$

$\Rightarrow x = 2y - 7$ --- (ii)

Substituting Eq. (ii) in (i), we obtain

$\Rightarrow (2y - 7)^2 + y^2 - 2(2y - 7) - 10y + 1 = 0$ [02 Marks]

$\Rightarrow 4y^2 - 28y + 49 + y^2 - 4y + 14s - 10y + 1 = 0$

$\Rightarrow 5y^2 - 42y + 64 = 0$

$\Rightarrow y = 2$ and $y = \frac{32}{5}$ [02 Marks]

Substitute the values of y in Eq. (ii), we get

$\Rightarrow x = -3$ and $x = \frac{29}{5}$

The endpoints of the chord are $A(-3,2)$ and $B\left(\frac{29}{5}, \frac{32}{5}\right)$ [02 Marks]

Now, we shall find the coordinates of the midpoint of the chord,

Midpoint of the chord = $\left(\frac{-3+\frac{29}{5}}{2}, \frac{2+\frac{32}{5}}{2}\right)$

$= \left(\frac{\frac{14}{5}, \frac{42}{5}}{2}\right) = \left(\frac{7}{5}, \frac{21}{5}\right)$ [02 Marks]
