Model Question Paper SSC-II

Mathematics(Science Group)

(2nd Set)SOLUTION

SECTION-A

1	**B**	**2**	**A**	**3**	**D**	**4**	**C**	**5**	**B**	**6**	**D**	**7**	**A**	**8**	**D**
9	**B**	**10**	**B**	**11**	**B**	**12**	**A**	**13**	**B**	**14**	**C**	**15**	**B**		

SECTION-B

Question 2

(i) Given that $(x-2)$ and $(x+2)$ are the 2 roots of given $P(x)=x^3-4mx^2-2nx+1=0$
Since $(x-2)$ is a root of the polynomial, so $x-2=0 \Longrightarrow x=2$
Using Synthetic division.

$$\begin{array}{c|ccc|c}
 & 1 & -4m & -2n & 1 \\
2 & & 2 & (-8m+4) & (-16m-4n+8) \\
\hline
 & 1 & (-4m+2) & (-8m-2n+4) & (-16m-4n+9)
\end{array}$$

Here $9-16m-4n=0 \quad \Longrightarrow \quad 16m+4n=9 \quad \longrightarrow \; eqn(1)$

Since $(x+2)$ is a root of the polynomial, so $x+2=0 \quad \Longrightarrow x=-2$

Using Synthetic division.

$$\begin{array}{c|ccc|c}
 & 1 & -4m & -2n & 1 \\
-2 & & -2 & 4+8m & (-16m+4n-8) \\
\hline
 & 1 & (-4m-2) & (8m-2n+4) & (-16m+4n-7)
\end{array}$$

Here $-16m+4n-7=0 \quad \Longrightarrow \quad 16m-4n=-7 \quad \longrightarrow \; eqn(2)$

Adding $eqns\ (1)\ and\ (2)$,

$(16m+4n)+(16m-4n)=(9)+(-7)$

$32m=2 \quad \Longrightarrow m=\frac{1}{16}$

Substituting $m's$ value in $eqn(1)$

$16\left(\frac{1}{16}\right)+4n=9 \quad \Longrightarrow \quad 1+4n=9 \quad \Longrightarrow n=2$

(ii) $2x^{-2}-21=x^{-1}$
$2(x^{-1})^2-x^{-1}-21 \qquad \longrightarrow \; eqn(1)$
Let $\quad x^{-1}=y$
Substituting it in $eqn(1)$
$2y^2-y-21=0$

$2y^2 + 6y - 7y\ - 21\ = 0$

$2y(y + 3) - 7(y + 3) = 0$

$(y\ +\ 3)\ (2y\ - 7\ )\ = 0$

Either $y\ =\ -3$ or $y\ =\ \frac{7}{2}$

By back substitution,

$x^{-1}\ =\ -3$ or $x^{-1}\ =\ \frac{7}{2}$

$x = -\frac{1}{3}$ or $x = \frac{2}{7}$

$Solution\ Set:\ \left\{-\frac{1}{3}, \frac{2}{7}\right\}$

(iii) $(7 - 5x,\ 3y\ +\ 2)\ =\ (y + 1\ ,\ x -\ 2)$

Equating the x-coordinates

$7 - 5x\ =\ y + 1$

$-5x - y + 6 = 0$

Multiplying both sides by 3

$-15x - 3y + 18 = 0 \quad \longrightarrow \quad eqn(1)$

Equating the y- coordinates

$3y + 2 = x - 2$

$-x + 3y + 4 = 0$

$-x + 3y + 4 = 0 \quad \longrightarrow \quad eqn(2)$

Adding above equations

$-16x + 22 = 0 \qquad \Rightarrow x\ =\ \frac{22}{16} = \frac{11}{8}$

Substituting x's value in $\longrightarrow$ $eqn(2)$

$\Rightarrow\ -\frac{22}{16} + 3y + 4 = 0$

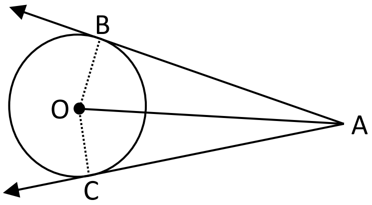
$3y + \frac{42}{16} = 0 \quad \Rightarrow \mathrm{y}\ =\ -\frac{14}{16} = -\frac{7}{8}$

Hence $x\ =\ \frac{11}{8}$ and $\mathrm{y}\ =\ -\frac{7}{8}$

(iv) **Given**: A circle with center O and A is any point outside the circle. $\overline{AB}\ and\ \overline{AC}$ are drawn two tangents from point A.

To Prove: $m\,\overline{AB}\ =\ m\,\overline{AC}$

Construction: Join O to A, B and C (as shown in figure)



Proof:

Statements	Reasons
In $\Delta AOB\ \leftrightarrow\ \Delta\,AOC,$	
$\overline{AO}\ \cong\ \overline{AO}$	Common
$\overline{OB}\ \cong\ \overline{OC}$	Radial Segment
$\angle\,\mathrm{ABO} \cong\ \angle\,ACO\ =\ 90°$	Radial segment ⊥Tangent line
$\Delta AOB\ \cong\ \Delta\,AOC$	H.S $\cong$ $H.S$
$\overline{AB}\ \cong\ \overline{AC}$	Corresponding sides of congruent triangles
$m\,\overline{AB}\ =\ m\,\overline{AC}$	

(v) **Given**: Two equal circles each of radius 3.5 cm. The distance between centres of the circles is 8cm.

Required: To draw Transverse Common Tangents.

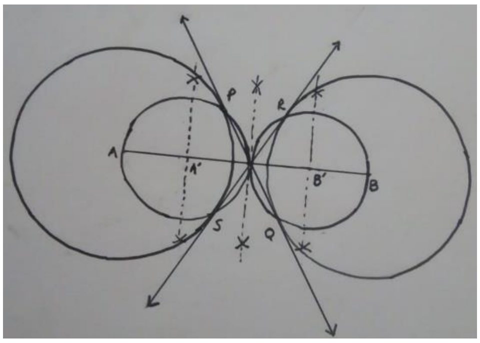
Steps of Construction:

i. Draw $m\overline{AB} = 8cm.$

ii. Draw two circles each of radius 3.5 cm with centres at A and B.

iii. Draw perpendicular bisector of $\overline{AB}$ at O.

iv. Draw perpendicular bisector of $\overline{AO}$ at $\acute{A}$.



v. Draw a circle of radius $\overline{A\acute{A}}$ with centre at $\acute{A}$ intersecting the left circle at points P and S.

vi. Draw perpendicular bisector of $\overline{BO}$ at $\acute{B}$.

vii. Draw a circle of radius $\overline{B\acute{B}}$ with centre at $\acute{B}$ intersecting the right circle at points R and Q.

viii. Join R to S and P to Q.

ix. $\overleftrightarrow{PQ}$ and $\overleftrightarrow{RS}$ are the required Transverse Common Tangents.

(vi) If α, β are the roots of $4z^2 + 17z + k = 0$, then

Sum of roots: $\alpha + \beta = -\frac{17}{4} \Longrightarrow \beta = -\frac{17}{4} - \alpha \longrightarrow eqn(1)$

Product of roots: $\alpha\beta = \frac{k}{4} \longrightarrow eqn(2)$

Given that: $2\alpha + 3\beta = 35$

Using $eqn(1)$

$$2\alpha + 3\left(-\frac{17}{4} - \alpha\right) = 35$$

$$2\alpha - \frac{51}{4} - 3\alpha = 35$$

$$\alpha = -\frac{51}{4} + 35 = \frac{89}{4}$$

Substituting it in $eqn(1)$

$$\beta = -\frac{17}{4} - \frac{89}{4} = -\frac{106}{4} = -\frac{53}{2}$$

Putting the values of α, β in $eqn(2)$

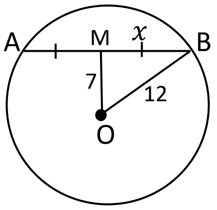
$$\alpha\beta = \frac{k}{4}$$

$$\left(\frac{89}{4}\right)\left(-\frac{53}{2}\right) = \frac{k}{4}$$

$$k = -\frac{4717}{2} = -2358.5$$

(vii) Let $\overline{AB}$ be the chord of a circle having centre at O.

Given that: Radius $\overline{OB} = 12cm$ and $\overline{OM} = 7cm$. Where $\overline{OM}$ is the perpendicular bisector of $\overline{AB}$



In right triangle BOM (By Pythagoras theorem)

$$\overline{OB}^2 = \overline{OM}^2 + \overline{BM}^2$$

$$12^2 = 7^2 + x^2$$

$$x^2 = 144 - 49 = 95$$

$$x = \sqrt{95} \text{ cm}$$

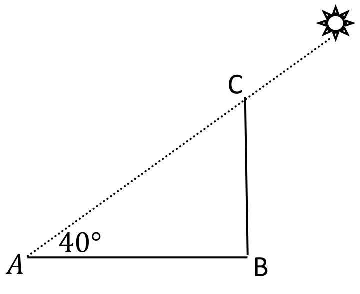
$$Chord\ \boldsymbol{\overline{AB}} = 2x = 2\sqrt{95}\ cm$$

(viii) In $\Delta\, ABC$

Height of the tree: $m\overline{BC} = 24\ ft$

Angle of elevation: $m\angle C = 40°$

Length of shadow: $m\overline{AC} = x$



$$\tan m\angle C = \frac{m\overline{BC}}{m\overline{AC}}$$

$$\tan 40° = \frac{24}{m\overline{AC}}$$

$$m\overline{AC} = \frac{24}{tan\ 40} = 28.57\ feet$$

(ix) $\frac{x^3}{x^2-x-2} = (x + 1) + \frac{3x+2}{x^2-x-2}$

$\frac{x^3}{x^2-x-2} = (x + 1) + \frac{3x+2}{(x-2)(x+1)} \quad \longrightarrow eqn(1)$

Resolving $\frac{3x+2}{(x-2)(x+1)}$ into Partial Fractions

$\frac{3x+2}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1} \quad \longrightarrow eqn(2)$

$3x + 2 = A(x + 1) + B(x - 2) \quad \longrightarrow eqn(3)$

Put $x = -1$ in eq (3)

$3(-1) + 2 = B(-1-2)$

$\mathbf{B} = \frac{1}{3}$

Put $x = 2$ in eq (3)

$3(2) + 2 = A(2+1)$

$\mathbf{A} = \frac{8}{3}$

Putting the values of A and B in $eqn(2)$

$$\frac{3x+2}{(x-2)(x+1)} = \frac{1}{3(x-2)} + \frac{8}{3(x+1)}$$

Putting this value in $eqn(1)$

$$\frac{x^3}{x^2-x-2} = (x + 1) + \frac{1}{3(x-2)} + \frac{8}{3(x+1)}$$

$$\begin{array}{r|l|l} x^2 - x - 2 & x^3 & x+1 \\ & \pm x^3 \mp x^2 \mp 2x & \\ \hline & x^2 + 2x & \\ & \pm x^2 \mp x \mp 2 & \\ \hline & 3x + 2 & \end{array}$$

Consider $x^2 - x - 2$

$= x^2 - 2x + x - 2$

$= x(x-2) + 1(x-2)$

$= (x-2)(x+1)$

(x) $x \propto \frac{1}{y}$ *and* $x \propto zt$

$x = \frac{k\,zt}{y}$ Where k is the constant of proportionality

$xy = kzt \quad \longrightarrow eqn(1)$

Put $x = 8, y = \frac{7}{2}, z = 14, t = 5$ in $eqn(1)$

$xy = kzt$

$8\left(\frac{7}{2}\right) = k(14)(5) \implies 28 = 70k \implies \boldsymbol{k} = \frac{\mathbf{2}}{\mathbf{5}}$

Put $x = 20,\ y = \frac{9}{2}, z = 23$ *and* $k = \frac{2}{5}$ in $eqn(1)$

$xy = kzt$

$20\left(\frac{9}{2}\right) = \frac{2}{5}(23)t \implies 90(5) = 2(23)t \implies \boldsymbol{t} = \frac{\mathbf{225}}{\mathbf{23}}$

(xi) $A = \{1, 2, 3, 4\}$ and $B = \{5, 6, 8\}$

a. $A \times B = \{(1,5), (1,6), (1,8)\ (2,5), (2,6), (2,8), (3,5), (3,6), (3,8), (4,5), (4,6), (4,8)\}$

b. $R = \{(x,y) |\ y = 2x\} = \{(3,6), (4,8)\}$

c. Domain of $R = \{3, 4\}$ Range of $R = \{6, 8\}$

(xii) In ΔXYZ, $m\overline{XY} = 8\sqrt{2}\,cm$, $m\overline{YZ} = 12\ cm$, and $m\angle XYZ = 135°$ (Obtuse angle)

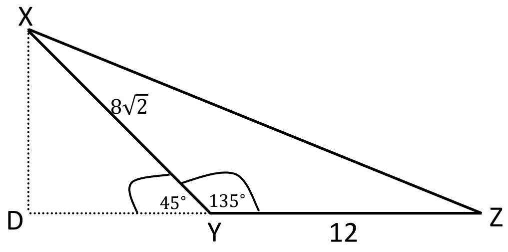
Using $(m\overline{XZ})^2 = (m\overline{XY})^2 + (m\overline{YZ})^2 + 2\,(m\overline{YZ})(m\overline{YD}) \quad \longrightarrow eqn(1)$

In $\Delta\, XYD$

$Cos\ 45° = \frac{m\overline{YD}}{m\overline{XY}}$

$m\overline{YD} = m\overline{XY}\ Cos\ 45°$

$m\overline{YD} = 8\sqrt{2}\ \frac{1}{\sqrt{2}} = 8cm$



Putting values in $eqn(1)$

$(m\overline{XZ})^2 = (8\sqrt{2})^2 + (12)^2 + 2\,(12)(8)$

$(m\overline{XZ})^2 = 464$

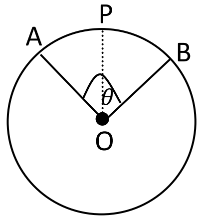
$\boldsymbol{m\overline{XZ} = 21.5cm}$

(xiii) Consider a circle of radius 'r', and an arc of length 1 unit, subtending an angle θ at O,
Area of circle = πr^2, Angle of circle = 2π , and Angle of sector = θ radians.
By the elementary geometry

$$\frac{Area\ of\ sector\ AOBP}{Area\ of\ circle} = \frac{Angle\ of\ sector}{Angle\ of\ Circle}$$

$$\frac{A}{\pi r^2} = \frac{\theta}{2\pi}$$

$$A = \frac{1}{2}\theta\gamma^z$$



(xiv) For five month moving average,

Month	Attendance	5-month moving Average
January	70	
February	82	$\frac{70+82+85+85+83}{5} = 81$
March	85	$\frac{82+85+85+83+78}{5} = 82.6$
April	85	$\frac{85+85+83+78+75}{5} = 81.2$
May	83	$\frac{85+83+78+75+80}{5} = 80.2$
June	78	-
July	75	-
August	80	-

SECTION-C

Q3 $U = \{1, 2, 3\ \dots, 20\}$; $A = \{2, 4, 6 \dots, 20\}$; $B = \{2, 3, 5 \dots, 19\}$

De-Morgan's Laws are as follows:

i. $(A \cup B)^c = A^c \cap B^c$

Proof:
$A \cup B = \{2, 4, 6 \dots, 20\} \cup \{2, 3, 5 \dots, 19\}$
$A \cup B = \{2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20\}$
$(A \cup B)^c = U - (A \cup B) = \{1,9,15\} \quad \longrightarrow eqn(1)$

$A^c = U - A = \{1,3,5,7,9,11,13,15,17,19\}$
$B^c = U - B = \{1, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20\}$
$A^c \cap B^c = \{1, 9, 15\} \quad \longrightarrow eqn(2)$
From $eqns(1 \& 2)$
$(A \cup B)^c = A^c \cap B^c$

ii. $(\mathrm{A} \cap \mathrm{B})^c = A^c \cup B^c$

Proof:

$A \cap B = \{2, 4, 6 \dots, 20\} \cap \{2,3,5 \dots, 19\} = \{2\}$
$(\mathrm{A} \cap \mathrm{B})^c = U - (A \cap B)$
$(\mathrm{A} \cap \mathrm{B})^c = \{1, 3, 4, 5, 6, 7 \dots, 20\} \quad \longrightarrow eqn(3)$

$A^c = U - A = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$
$B^c = U - B = \{1, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20\}$
$A^c \cup B^c = \{1, 3, 4, 5, 6, 7 \dots, 20\} \quad \longrightarrow eqn(4)$
From $eqns(3 \& 4)$
$(A \cap B)^c = A^c \cup B^c$

Q4 $a^2 + b^2 = 20 \quad \longrightarrow eqn(1)$

$3a^2 - 2ab - b^2 = 0$

$3a^2 - 3ab + ab - b^2 = 0$

$3a(a-b) + b\,(a-b) = 0$

$(3a + b)(a-b) = 0$

$3a + b = 0$ or $a - b = 0$

$a = -\frac{b}{3}$ or $a = b$

When $a = -\frac{b}{3}$

$eqn(1) \longrightarrow \frac{b^2}{9} + b^2 = 20$

$10b^2 = 180$

$b^2 = 18$

$b = \pm 3\sqrt{2}$

Taking $a = -\frac{b}{3}$

$a = -\frac{\pm 3\sqrt{2}}{3} = \mp\sqrt{2}$

When $a = b$

$eqn(1) \longrightarrow b^2 + b^2 = 20$

$2b^2 = 20$

$b^2 = 10$

$b = \pm\sqrt{10}$

Taking $a = b$

$a = \pm\sqrt{10}$

Solution Set: $\{(\mp\sqrt{2}, \pm 3\sqrt{2}), (\pm\sqrt{10}, \pm\sqrt{10})\}$

Q5 i. $(tan\theta + Cot\,\theta)(Cos\theta + Sin\theta) = Sec\theta + Cosec\theta$

$\left(\frac{Sin\theta}{Cos\theta} + \frac{Cos\theta}{Sin\theta}\right)(Cos\theta + Sin\theta) = Sec\theta + Cosec\theta$

$\left(\frac{Sin^2\theta + Cos^2\theta}{SIn\theta\, Cos\theta}\right)(Cos\theta + Sin\theta) = Sec\theta + Cosec\theta$

$\frac{1}{Sin\theta\, Cos\theta}(Cos\theta + Sin\theta) = Sec\theta + Cosec\theta$

$\frac{1}{Sin\theta} + \frac{1}{Cos\theta} = Sec\theta + Cosec\theta$

$Sec\theta + Cosec\theta = Sec\theta + Cosec\theta$

Hence proved

ii. $\frac{Cos\theta - Sin\theta}{Cot^2\theta - 1} = \frac{Sin^2\theta}{Cos\theta + Sin\theta}$

$\frac{Cos\theta - Sin\theta}{\frac{Cos^2\theta}{Sin^2\theta} - 1} = \frac{Sin^2\theta}{Cos\theta + Sin\theta}$

$\frac{Cos\theta - Sin\theta}{\frac{Cos^2\theta - Sin^2\theta}{Sin^2\theta}} = \frac{Sin^2\theta}{Cos\theta + Sin\theta}$

$\frac{Sin^2\theta(Cos\theta - Sin\theta)}{Cos^2\theta - Sin^2\theta} = \frac{Sin^2\theta}{Cos\theta + Sin\theta}$

$\frac{Sin^2\theta(Cos\theta - Sin\theta)}{(Cos\theta - Sin\theta)(Cos\theta + Sin\theta)} = \frac{Sin^2\theta}{Cos\theta + Sin\theta}$

$\frac{Sin^2\theta}{Cos\theta + Sin\theta} = \frac{Sin^2\theta}{Cos\theta + Sin\theta}$

Hence proved

Q6 $\sqrt{x^2 + 3x + 5} + \sqrt{x^2 + 3x - 2} = 7$

$\sqrt{x^2 + 3x + 5} = 7 - \sqrt{x^2 + 3x - 2}$

Squaring both sides

$x^2 + 3x + 5 = 49 + (x^2 + 3x - 2) - 14\sqrt{x^2 + 3x - 2}$

$42 = 14\sqrt{x^2 + 3x - 2}$

$\sqrt{x^2 + 3x - 2} = 3$

Squaring both sides

$x^2 + 3x - 2 = 9$

$x^2 + 3x - 11 = 0$

Using quadratic formula

$x = \frac{-3 \pm \sqrt{9+44}}{2} = \frac{-3 \pm \sqrt{53}}{2}$

Check:

$\sqrt{x^2 + 3x + 5} + \sqrt{x^2 + 3x - 2} = 7$

At $x = \frac{-3+ \sqrt{53}}{2}$

$$\sqrt{\left(\frac{-3 + \sqrt{53}}{2}\right)^2 + 3\left(\frac{-3 + \sqrt{53}}{2}\right) + 5} + \sqrt{\left(\frac{-3 + \sqrt{53}}{2}\right)^2 + 3\left(\frac{-3 + \sqrt{53}}{2}\right) - 2} = 7$$

$$\sqrt{\frac{62 - 6\sqrt{53}}{4} + \frac{-9 + 3\sqrt{53}}{2} + 5} + \sqrt{\frac{62 - 6\sqrt{53}}{4} + \frac{-9 + 3\sqrt{53}}{2} - 2} = 7$$

$$\sqrt{\frac{62 - 6\sqrt{53}}{4} + \frac{-18 + 6\sqrt{53}}{4} + 5} + \sqrt{\frac{62 - 6\sqrt{53}}{4} + \frac{-18 + 6\sqrt{53}}{4} - 2} = 7$$

$\sqrt{11 + 5} + \sqrt{11 - 2} = 7$

$7 = 7$

At $x = \frac{-3- \sqrt{53}}{2}$

$$\sqrt{\left(\frac{-3 - \sqrt{53}}{2}\right)^2 + 3\left(\frac{-3 - \sqrt{53}}{2}\right) + 5} + \sqrt{\left(\frac{-3 - \sqrt{53}}{2}\right)^2 + 3\left(\frac{-3 - \sqrt{53}}{2}\right) - 2} = 7$$

$$\sqrt{\frac{62 + 6\sqrt{53}}{4} - \frac{9 + 3\sqrt{53}}{2} + 5} + \sqrt{\frac{62 + 6\sqrt{53}}{4} - \frac{9 + 3\sqrt{53}}{2} - 2} = 7$$

$$\sqrt{\frac{62 + 6\sqrt{53}}{4} - \frac{18 + 6\sqrt{53}}{4} + 5} + \sqrt{\frac{62 + 6\sqrt{53}}{4} - \frac{18 + 6\sqrt{53}}{4} - 2} = 7$$

$\sqrt{11 + 5} + \sqrt{11 - 2} = 7$

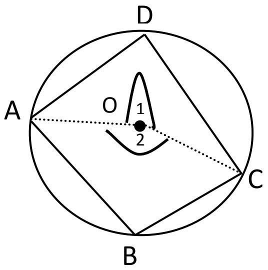
$7 = 7$

Solution Set: $\left\{\frac{-3+ \sqrt{53}}{2}\right\}$

Q7 **Given**: A quadrilateral $ABCD$ is inscribed in a circle with centre at O.

To Prove: $m\angle ABC + m\angle ADC = 180^\circ$ and $m\angle BCD + m\angle BAD = 180^\circ$

Figure:



Construction: Join O to A and C.

Proof: $\angle AOC$ is a central angle of the arc ABC and

$\angle ABC$ is an inscribed angle at B on the remaining part of the circle.

Statements	Reasons
$m\angle 1 = 2\,m\angle ABC \longrightarrow eqn(1)$ $m\angle 2 = 2\,m\angle ADC \longrightarrow eqn(2)$	The angle which an arc of a circle subtends at the Center is twice of the angle subtended at any point on the remaining part of the circumference.
$m\angle 1 + m\angle 2 = 2\,[m\angle ABC + m\angle ADC]$	Same as above
$m\angle 1 + m\angle 2 = 360^\circ$	Adding $eqns(1\&2)$
$2\,[m\angle ABC + m\angle ADC] = 360^\circ$	
$[m\angle ABC + m\angle ADC] = 180^\circ$	
Similarly $m\angle BCD + m\angle BAD = 180^\circ$	Hence proved