Model Question Paper SSC-I

Mathematics(Science Group)

(2^{nd} Set)SOLUTION

SECTION-A

1	D	2	A	3	D	4	A	5	D	6	A	7	A	8	A
9	A	10	B	11	A	12	A	13	D	14	A	15	D		

SECTION-B

Question-2(i)

BC= $\begin{bmatrix} -5 & -3 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} -18 & -11 \\ 7 & 4 \end{bmatrix}$

A (BC)= $\begin{bmatrix} 0 & 2 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} -18 & -11 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 14 & 8 \\ -61 & -37 \end{bmatrix}$

AB= $\begin{bmatrix} 0 & 2 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} -5 & -3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ -17 & -10 \end{bmatrix}$

(AB) C= $\begin{bmatrix} 4 & 2 \\ -17 & -10 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 14 & 8 \\ -61 & -37 \end{bmatrix}$

From above A (BC) = (AB) C

Question-2(ii)

Let $x = \frac{\sqrt[3]{46.3}(0.05)^2}{\sqrt{8.54}}$

Taking log of both sides

$logx = log\sqrt[3]{46.3} + log(0.05)^2 - log\sqrt{8.54}$

$logx = log(46.3)^{1/3} + log(0.05)^2 - log(8.54)^{1/2}$

$logx = \frac{1}{3}log46.3 + 2log0.05 - \frac{1}{2}log8.54$

$logx = \frac{1}{3}(1.6660) + 2(\overline{2}.6990) - \frac{1}{2}(0.9315)$

$logx = 0.5553 + 2(-2 + 0.6990) - 0.4658$

$logx = 0.5553 - 4 + 1.3980 - 0.4658$

$logx = -2.5125 = \overline{2}.5125$

$x = antilog\overline{2}.5125$

$x = 0.03256$

Question-2(iii)

$$\left(\frac{15m^3n^{-2}p^{-1}}{25m^{-2}n^{-9}}\right)^{-3}$$

$$= \left(\frac{3m^{3+2}n^{-2+9}p^{-1}}{5}\right)^{-3}$$

$$= \left(\frac{3m^5n^7}{5p}\right)^{-3} = \left(\frac{5p}{3m^5n^7}\right)^3$$

$$= \left(\frac{5^3p^3}{3^3m^{5\times3}n^{7\times3}}\right)$$

$$= \left(\frac{125p^3}{27m^{15}n^{21}}\right)$$

Question-2(iv)

$(1+i)^3(x+yi) = (4+5i)$

$(1+3i+3i^2+i^3)(x+yi) = 4+5i$

$(1+3i-3-i)(x+yi) = 4+5i$

$(2i-2)(x+yi) = 4+5i$

$2xi + 2yi^2 - 2x - 2yi = 4+5i$

$2xi - 2y - 2x - 2yi = 4+5i$

$(-2x-2y) + (2x-2y)i = 4+5i$

Equating the real and imaginary parts

$-2x - 2y = 4 \longrightarrow eqn - I$ $\qquad$ $2x - 2y = 5 \longrightarrow eqn - II$

Adding equations I and II

$-2x - 2y + 2x - 2y = 4 + 5$

$-4y = 9 \qquad \Rightarrow y = -\frac{9}{4}$

Subtracting equations II from I

$-2x - 2y - 2x + 2y = 4 - 5$

$-4x = -1 \qquad \Rightarrow x = \frac{1}{4}$

Question-2(v)

$$x - \frac{1}{x} = 7$$

$$\left(x - \frac{1}{x}\right)^3 = 7^3$$

$$x^3+\frac{1}{x^3}-3\left(x-\frac{1}{x}\right)=343$$

$$x^3+\frac{1}{x^3}-3(7)=343 \qquad \because x-\frac{1}{x}=7$$

$$x^3+\frac{1}{x^3}=343+21$$

$$x^3+\frac{1}{x^3}=364$$

Question-2(vi)

(a) $x=-3+\sqrt{2}$

$$\frac{1}{x}=\frac{1}{-3+\sqrt{2}}$$

$$\frac{1}{x}=\frac{1}{-3+\sqrt{2}}\times\frac{-3-\sqrt{2}}{-3-\sqrt{2}}$$

$$\frac{1}{x}=\frac{-3-\sqrt{2}}{(-3)^2-\left(\sqrt{2}\right)^2}=\frac{-3-\sqrt{2}}{7}=-\frac{3+\sqrt{2}}{7}$$

(b) $x+\frac{1}{x}=-3+\sqrt{2}-\frac{3+\sqrt{2}}{7}$

$$x+\frac{1}{x}=\frac{-21+7\sqrt{2}-3-\sqrt{2}}{7}=\frac{-24+6\sqrt{2}}{7}$$

(c) $x-\frac{1}{x}=-3+\sqrt{2}+\frac{3+\sqrt{2}}{7}$

$$x-\frac{1}{x}=\frac{-21+7\sqrt{2}+3+\sqrt{2}}{7}=\frac{-18+8\sqrt{2}}{7}$$

(d) $x^2+\frac{1}{x^2}=\left(-3+\sqrt{2}\right)^2+\left(-\frac{3+\sqrt{2}}{7}\right)^2$

$$x^2+\frac{1}{x^2}=9+2-6\sqrt{2}+\frac{9+2+6\sqrt{2}}{49}$$

$$x^2+\frac{1}{x^2}=11-6\sqrt{2}+\frac{11+6\sqrt{2}}{49}$$

$$x^2+\frac{1}{x^2}=\frac{550-288\sqrt{2}}{49}$$

Question-2(vii)

$$2x^2+7x+\frac{6}{x}$$

$2x^2$	$4x^4+28x^3+49x^2+24x+84+\frac{36}{x^2}$ $\pm 4x^4$
$4x^2+7x$	$28x^3+49x^2+24x+84+\frac{36}{x^2}$ $\pm 28x^3 \pm 49x^2$

$4x^2 + 14x + \frac{6}{x}$ $\quad$ $24x + 84 + \frac{36}{x^2}$

$\pm 24x \pm 84 \pm \frac{36}{x^2}$

0

$$\sqrt{4x^4 + 28x^3 + 49x^2 + 24x + 84 + \frac{36}{x^2}} = \pm\left(2x^2 + 7x + \frac{6}{x}\right)$$

Question-2(viii)

$x^2 + 4x - 12 = x^2 + 6x - 2x - 12 = x(x+6) - 2(x+6) = (x+6)(x-2)$

$x^2 - 4 = (x+2)(x-2)$

$x^3 - 2^3 = (x-2)(x^2 + 2x + 4)$

H.C.F= $(x-2)$

Question-2(ix)

$P(x) = x^3 - 2x^2 - 5x + 6$

At $x = 1$

$P(1) = 1 - 2 - 5 + 6 = 0$

$x - 1$ is a factor of $P(x)$

$x - 1$) $x^3 - 2x^2 - 5x + 6$ ($x^2 - x - 6$

$\pm x^3 \mp x^2$

$-x^2 - 5x + 6$

$\mp x^2 \pm x$

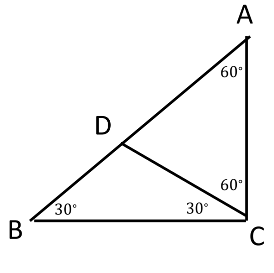
$-6x + 6$

$\mp 6x \pm 6$

$P(x) = (x-1)(x^2 - x - 6)$

$P(x) = (x-1)(x-3)(x+2)$

Question-2(x)



Given: In $\triangle ABC$, $\angle C = 90°$, $\angle A = 60°$

To Prove: $\overline{BC} = \frac{1}{2}\overline{AB}$

Construction: At C construct $\angle BCD = 30°$. Let $\overline{CD}$ cuts $\overline{AB}$ at D.

Proof:

<u>Statements</u> $\qquad$ <u>Reasons</u>

In $\triangle$ADC, $\angle A = 60°$	Given
$\angle CDA = 60°$	$m\angle BCD + m\angle CDA = 90°$
$\therefore\triangle$ADC is equilateral	
$\overline{AC} = \overline{CD} = \overline{AD}$	
$\overline{AB} = \overline{BD} + \overline{AD}$	$\triangle BDC$ is isosceles
$\overline{AB} = \overline{DC} + \overline{AC}$	$\overline{BD} = \overline{DC}$ & $\overline{AD} = \overline{AC}$
$\overline{AB} = \overline{AC} + \overline{AC}$	
$\overline{AB} = 2\overline{AC}$	
$\overline{AC} = \frac{1}{2}\overline{AB}$	

Question-2(xi)

A(1, 1), B(3, 1), C(4, 3)

$\left|\overline{AB}\right|^2 = (3-1)^2 + (1-1)^2 = 2$

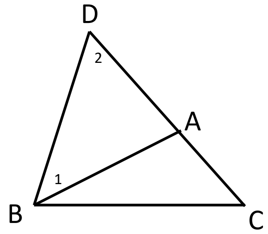
$\left|\overline{BC}\right|^2 = (4-3)^2 + (3-1)^2 = 5$

$\left|\overline{AC}\right|^2 = (4-1)^2 + (3-1)^2 = 13$

Since $\left|\overline{AB}\right|^2 + \left|\overline{BC}\right|^2 = 2 + 5 = 7 \neq \left|\overline{AC}\right|^2$

Hence ABC is not a right angled triangle.

Question-2(xii)



Given: In $\triangle$ABC

To Prove: (i) $m\overline{AB} + m\overline{AC} > m\overline{BC}$

(ii) $m\overline{AB} + m\overline{BC} > m\overline{AC}$

(iii) $m\overline{BC} + m\overline{AC} > m\overline{AB}$

Construction: Take a point D on $\overline{CA}$ such that $\overline{AD} = \overline{AB}$. Join B to D.

Proof:

Statements		Reasons
In $\triangle$ABD		Given
$\angle 1 \cong \angle 2$		Construction
$m\angle DBC > m\angle 1$	eqn(i)	$m\angle DBC + m\angle 1 + m\angle ABC$
$m\angle DBC > m\angle 2$	eqn(ii)	from (i) and (ii)
In $\triangle DBC$	eqn(iii)	

$m\overline{DC} > m\overline{BC}$ from (iii)

$m\overline{AD} + m\overline{AC} > m\overline{BC}$ $m\overline{CD} = m\overline{AD} + m\overline{AC}$

Hence $m\overline{AB} + m\overline{AC} > m\overline{BC}$ $m\overline{AD} = m\overline{AB}$

Similarly

$m\overline{AB} + m\overline{BC} > m\overline{AC}$

$m\overline{BC} + m\overline{AC} > m\overline{AB}$

Question-2(xiii)

A$(2,4)$, B$(4,4)$, C$(-1,3)$, D$(-3,3)$

$|\overline{AB}| = \sqrt{(4-2)^2 + (4-4)^2} = 2$

$|\overline{DC}| = \sqrt{(-1+3)^2 + (3-3)^2} = 2$

$|\overline{AD}| = \sqrt{(-3-2)^2 + (3-4)^2} = \sqrt{26}$

$|\overline{BC}| = \sqrt{(-1-4)^2 + (3-4)^2} = \sqrt{26}$

Since $|\overline{AB}| = |\overline{DC}|$ and $|\overline{AD}| = |BC|$

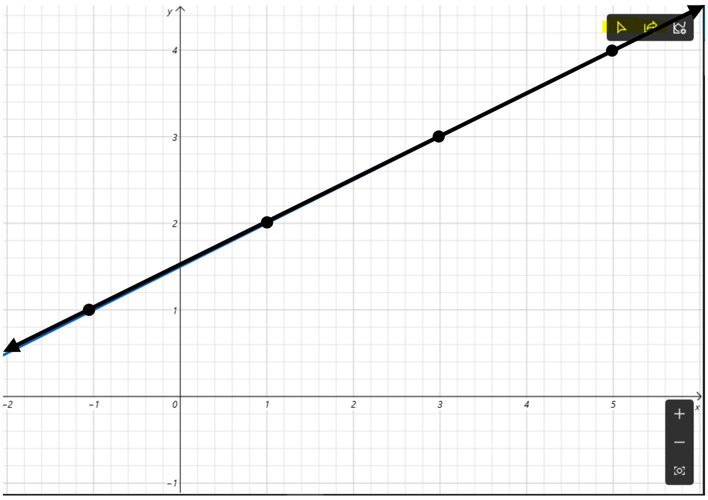
i.e. opposite sides of the quadrilateral ABCD are equal.

Hence ABCD is a parallelogram.

Question-2(xiv)

$2y - x - 3 = 0$

x	-1	1	3	5
y	1	2	3	4



SECTION-C

Q3. (a) $\left|\frac{x+8}{12}\right| = \frac{x-1}{5}$

$\frac{x+8}{12} = +\left(\frac{x-1}{5}\right)$ $\frac{x+8}{12} = -\left(\frac{x-1}{5}\right)$

$12(x-1) = 5(x+8)$ $12(x-1) = -5(x+8)$

$12x - 12 = 5x + 40$ $12x - 12 = -5x - 40$

$12x - 5x = 40 + 12$ $12x + 5x = 12 - 40$

$7x = 52$ $17x = -28$

$x = \frac{52}{7}$ $x = -\frac{28}{17}$

Solution set= $\left\{\frac{52}{7}, -\frac{28}{17}\right\}$

(b) $2 \leq \frac{2}{3} - 4x < 3 - 5x$

Multiply by 3

$6 \leq 2 - 12x < 9 - 15x$

$6 \leq 2 - 12x$ $2 - 12x < 9 - 15x$

$12x \leq 2 - 6$ $15x - 12x < 9 - 2$

$12x \leq -4$ $3x < 7$

$x \leq -\frac{1}{3}$ $x < \frac{7}{3}$

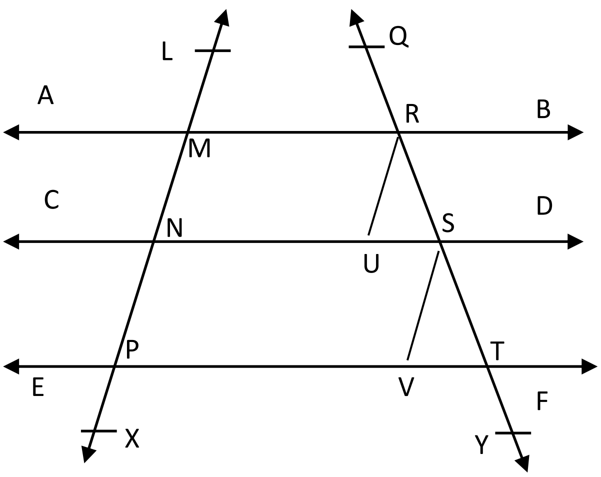
Solution Set = $\left\{x \middle| -\frac{1}{3} \geq x < \frac{7}{3}\right\}$

Q4

Statement: If three or more parallel lines make congruent segments on transversal, they also intercept congruent segments on any other

line that cuts them.

Figure:



Given: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$

The transversal $\overleftrightarrow{LX}$ intersects $\overleftrightarrow{AB}, \overleftrightarrow{CD}$ *and* $\overleftrightarrow{EF}$ at the points M, N and P respectively, such that $\overline{MN} \cong \overline{NP}$. The transversal $\overleftrightarrow{QY}$ intersects them at points R, S and T respectively.

To Prove: $\overline{RS} = \overline{ST}$

Construction: From R, draw $\overline{RU} \parallel \overline{LX}$, which meets $\overline{CD}$ at U. From S, draw $\overline{SV} \parallel \overline{LX}$ which meets $\overline{EF}$ at V. As shown in the figure let the angles be labelled as $\angle 1, \angle 2, \angle 3$ and $\angle 4$.

Proof:

Statements	**Reasons**
MNUR is a parallelogram.	$\overline{RU} \parallel \overline{LX}$, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$
$\overline{MN} \cong \overline{RU} \quad \longrightarrow (i)$	Opposite sides of a $\parallel gram$
Similarly,	
$\overline{NP} \cong \overline{SV} \quad \longrightarrow (ii)$	
But $\overline{MN} \cong \overline{NP} \longrightarrow (iii)$	Given
$\therefore \overline{RU} \cong \overline{SV}$	From (i), (ii) and (iii)
Also $\overline{RU} \parallel \overline{SV}$	each is $\parallel \overleftrightarrow{LX}$
$\therefore \angle 1 \cong \angle 2$ and $\quad \angle 3 \cong \angle 4$	Corresponding angles
In $\Delta RUS \leftrightarrow \Delta SVT$	
$\overline{RU} \cong \overline{SV}$	Proved
$\angle 1 \cong \angle 2$	Proved
$\angle 3 \cong \angle 4$	Proved
$\Delta RUS \cong \Delta SVT$	S.A.A≅S.A.A
Hence $\overline{RS} = \overline{ST}$	Corresponding sides of congruent triangles.

Q5.

Let cost of chair $= x$

Let cost of Table $= y$

According to First condition

$x = \frac{y}{2} + 3$

$\Longrightarrow 2x = y + 6 \quad \Longrightarrow 2x - y = 6 --- (1)$

According to 2nd condition

$3x + y = 54 --- (2)$

These equations can be written in form of matrices as ;

$$\begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 54 \end{bmatrix}$$

Let $A = \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}$,$X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 54 \end{bmatrix}$

$\therefore AX = B \Longrightarrow X = A^{-1}B --- (3)$

Now $|A| = \begin{vmatrix} 2 & -1 \\ 3 & 1 \end{vmatrix} = 2(1) - (-1 \times 3) = 5 \neq 0 --- (4)$

$Adj\ A = \begin{bmatrix} 1 & 1 \\ -3 & 2 \end{bmatrix} --- (5)$

$A^{-1} = \frac{Adj\ A}{|A|}$ ------(6)

Using Equation (4) and Equation (5) in Equation (6)

$$A^{-1} = \frac{1}{5}\begin{bmatrix} 1 & 1 \\ -3 & 2 \end{bmatrix} - - - (6)$$

Putting the values in Equation (3)

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 1 & 1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 6 \\ 54 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 6+54 \\ -18+108 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 60 \\ 90 \end{bmatrix} \quad \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 18 \end{bmatrix}$$

By definition of equal matrices their corresponding elements are equal

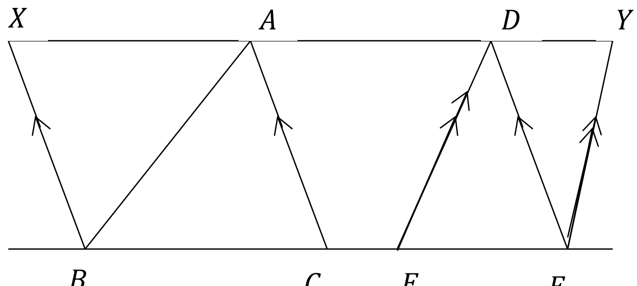
So x= 12 , y =18

∴ Cost of chair= Rs 12 and Cost of table=Rs 18

Q6.

Statement: Triangles on equal bases and of equal altitudes are equal in area.

Figure:



Given: As ABS, DEF on equal bases $\overline{BC}, \overline{EF}$ and having equal altitudes.

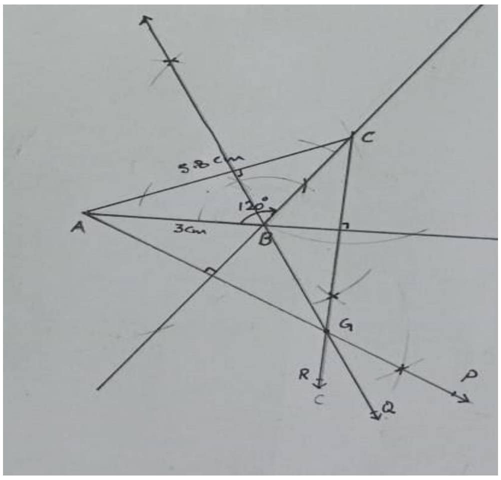
To prove: Area of ΔABC = Area of ΔDEF

Construction: Place the Δs ABC and DEF so that their equal bases $\overline{BC}$ and $\overline{EF}$ are on the same straight lines $BCEF$ and their vertices on the same side of it. Draw $\overline{BX} \parallel \overline{CA}$ and $\overline{FY} \parallel \overline{ED}$ meeting AD produces in X and Y, respectively.

Proof:

Statements	**Reasons**
$\Delta ABC, \Delta DEF$ are between the same parallels	Their altitudes are equal (Given).
∴ $XADY$ is parallel to $BCEF$	
∴ Area of ‖gm $BCAX$ = Area of ‖gm $EFYD \rightarrow (i)$	These ‖gms are on equal bases and between the same parallel lines.
But, Area of $\Delta ABC = \frac{1}{2}$(Area of ‖gm $BCAX$) $\rightarrow (ii)$	Diagonals of a ‖gm bisect it.
And Area of $\Delta DEF = \frac{1}{2}$(Area of ‖gm$EFYD$) $\rightarrow (iii)$	
∴ Area of ΔABC = Area of ΔDEF	From $(i), (ii)$ and (iii)

Q7 $m\overline{AB} = 3cm$, $m\angle B = 120°$, $m\overline{BC} = 5.8cm$



Construction Steps:

a. Draw $m\overline{AB} = 3cm$

b. Using pair of compasses to draw $m\angle B = 120°$.

c. With A as centre draw an arc of radius 5.8 cm that cuts $m\angle B$ at C.

d. ΔABC is completed.

d. Construct $\overline{AP}$ altitude from vertex A.

e. Construct $\overline{BQ}$ altitude from vertex B.

f. Construct $\overline{CR}$ altitude from vertex C.

g. The altitudes intersect at point G.

i.e. altitudes of ΔABC are concurrent.