SOLUTION QUESTION MODEL PAPER (3rd Set) SSC-I

MATHEMATICS

SECTION-A

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
B	C	A	A	A	C	B	A	C	C	B	C	A	C	B

SECTION-B

Question 2

(i) $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(a) $det(A) = (1)(3) - (2)(1) = 1$ $\rightarrow (01) mark$

$Adj(A) = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$ $\rightarrow (01) mark$

(b) $A(AdjA) = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 3-2 & -2+2 \\ 3-3 & -2+3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$ $\rightarrow (01) mark$

$(AdjA)A = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 3-2 & 6-6 \\ -1+1 & -2+3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$ $\rightarrow (01) mark$

Hence $A(AdjA) = (AdjA)A$

(ii) $(x - iy)(3 + 5i) = \overline{-6 - 24i}$

$(3x + 5y) + (5x - 3y)i = -6 + 24i$ $\rightarrow (01) mark$

$3x + 5y = -6 \qquad 5x - 3y = 24$ $\rightarrow (01) mark$

Multiplying equations by -5 and by 3 respectively then adding the resultant

$-15x - 25y + 15x - 9y = 30 + 72 \qquad \Longrightarrow y = -3$ $\rightarrow (01) mark$

Multiplying equations by 3 and by 5 respectively then adding the resultant

$9x + 15y + 25x - 15y = -18 + 120 \qquad \Longrightarrow x = 3$ $\rightarrow (01) mark$

(iii) $\log_4 (64)^{n+1} = \log_5 (625)^{n-1}$

$\log_4 (4)^{3(n+1)} = \log_5 (5)^{4(n-1)}$ $\rightarrow (01) mark$

$3(n+1) \log_4 4 = 4(n-1) \log_5 5$ $\rightarrow (01) mark$

$3(n+1) = 4(n-1)$ $\rightarrow (01) mark$

$n = 7$ $\rightarrow (01) mark$

(iv) $\frac{1}{x} = \sqrt{7} + \sqrt{6}$

$x = \frac{1}{\sqrt{7}+\sqrt{6}} \times \frac{\sqrt{7}-\sqrt{6}}{\sqrt{7}-\sqrt{6}} = \sqrt{7} - \sqrt{6}$ $\rightarrow (01) mark$

$x + \frac{1}{x} = (\sqrt{7} + \sqrt{6}) + (\sqrt{7} - \sqrt{6}) = 2\sqrt{7}$ $\rightarrow (01) mark$

$x - \frac{1}{x} = (\sqrt{7} + \sqrt{6}) - (\sqrt{7} - \sqrt{6}) = 2\sqrt{6}$ $\rightarrow (01) mark$

$\left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right) = (2\sqrt{7})(2\sqrt{6}) = 4\sqrt{42}$ $\rightarrow (01) mark$

(v) $P(x) = x^4 - 2x^3 - 11x^2 - 8x - 60$

At $x = -3$

$P(-3) = (-3)^4 - 2(-3)^3 - 11(-3)^2 - 8(-3) - 60 = 0$

Thus $(x+3)$ is a factor of $P(x)$. $\rightarrow (01) mark$

On dividing $P(x)$ by $(x+3)$

Other factor of $P(x) = (x^3 - 5x^2 + 4x - 20)$

$P(x) = (x^3 - 5x^2 + 4x - 20)(x+3)$ $\rightarrow (02) marks$

$P(x) = [x^2(x-5) + 4(x-5)](x+3)$

$P(x) = (x-5)(x^2+4)(x+3)$ $\rightarrow (01) mark$

$$
\begin{array}{r|l}
 & x^3 - 5x^2 + 4x - 20 \\
\hline
x+3 & x^4 - 2x^3 - 11x^2 - 8x - 60 \\
 & \pm x^4 \pm 3x^3 \\
 & -5x^3 - 11x^2 - 8x - 60 \\
 & \mp 5x^3 \mp 15x^2 \\
 & 4x^2 - 8x - 60 \\
 & \pm 4x^2 \pm 12x \\
 & -20x - 60 \\
 & \mp 20x \mp 60 \\
 & 0
\end{array}
$$

(vi) Let $P(x)$ be the required polynomial and $Q(x) = x^2 - 5x - 14$ the given polynomial with

HCF = $x - 7$ and LCM = $x^3 - 10x^2 + 11x + 70$

$P(x) = \frac{(HCF)(LCM)}{Q(x)}$ $\rightarrow (01) mark$

$P(x) = \frac{(x-7)(x^3 - 10x^2 + 11x + 70)}{(x^2 - 5x - 14)}$

$P(x) = \frac{(x-7)(x^3 - 10x^2 + 11x + 70)}{(x-7)(x+2)}$ $\rightarrow (01) mark$

$P(x) = \frac{(x^3 - 10x^2 + 11x + 70)}{(x+2)}$

$P(x) = x^2 - 3x - 10$ $\rightarrow (02) marks$

$$
\begin{array}{r|l}
 & x^2 - 3x - 10 \\
\hline
x-7 & x^3 - 10x^2 + 11x + 70 \\
 & \pm x^3 \mp 7x^2 \\
 & -3x^2 + 11x + 70 \\
 & \mp 3x^2 \pm 21x \\
 & -10x + 70 \\
 & \mp 10x \pm 70 \\
 & 0
\end{array}
$$

(vii) $\left|\frac{3x+9}{2x+1}\right| - 9 = 5$

$\left|\frac{3x+9}{2x+1}\right| = 14$

$\frac{3x+9}{2x+1} = 14$ $\rightarrow (01) mark$

$3x + 9 = 14(2x+1)$

$3x + 9 = 28x + 14$

$25x = -5$

$x = -\frac{1}{5}$ $\rightarrow (01) mark$

$\frac{3x+9}{2x+1} = -14$ $\rightarrow (01) mark$

$3x + 9 = -14(2x+1)$

$3x + 9 = -28x - 14$

$31x = -23$

$x = -\frac{23}{31}$ $\rightarrow (01) mark$

Solution Set = $\left\{-\frac{1}{5}, -\frac{23}{31}\right\}$

$(viii)$ $\frac{2}{3} \leq \frac{1+x}{6} \leq \frac{3}{4}$

$\frac{2}{3} \leq \frac{1+x}{6}$; $\frac{1+x}{6} \leq \frac{3}{4}$ $\rightarrow (01) mark$

$\frac{12}{3} \leq 1 + x$; $1 + x \leq \frac{18}{4}$ $\rightarrow (01) mark$

$4 - 1 \leq x$; $x \leq \frac{9}{2} - 1$ $\rightarrow (01) mark$

$3 \leq x$; $x \leq \frac{7}{2}$ $\rightarrow (01) mark$

Solution Set $= \left\{x | x \in R \wedge 3 \leq x \leq \frac{7}{2}\right\}$

(ix) $x + 2y = -4$

$y = -\frac{1}{2}(x + 4)$

x	2	0	−2	−4
y	−3	−2	−1	0

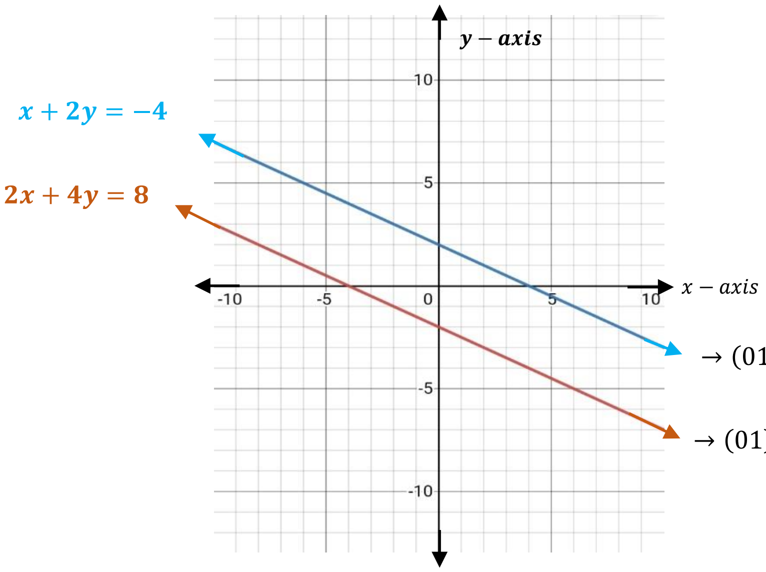
$\rightarrow (01) mark$

$2x + 4y = 8$

$y = -\frac{1}{2}(x - 4)$

x	4	2	0	−2
y	0	1	2	3

$\rightarrow (01) mark$



$\rightarrow (01) mark$

$\rightarrow (01) mark$

The given system of linear equations represents a pair of parallel straight lines on the graph.

Therefore $Solution\ Set = \{ \}$

(x) $P(3,3), Q(8,3), R(3,12)$

$|\overline{PQ}| = \sqrt{(8-3)^2 + (3-3)^2} = 5$ $\rightarrow (01) mark$

$|\overline{QR}| = \sqrt{(3-8)^2 + (12-3)^2} = \sqrt{106} = 10.3$ $\rightarrow (01) mark$

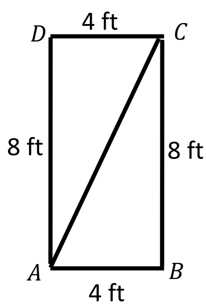
$|\overline{PR}| = \sqrt{(3-3)^2 + (12-3)^2} = 9$ $\rightarrow (01) mark$

$|\overline{PQ}| + |\overline{QR}| = 5 + 10.3 = 15.3 \neq |\overline{PR}|$ $\rightarrow (01) mark$

Therefore given points are not collinear.

(xi) Let ABCD represents a rectangular doorway

By Pythagoras Theorem



$m|\overline{AC}|^2 = m|\overline{AB}|^2 + m|\overline{BC}|^2$ $\rightarrow (01) mark$

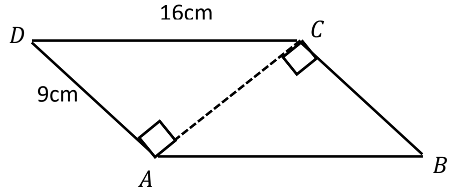
$m|\overline{AC}|^2 = 4^2 + 8^2$ $\rightarrow (01) mark$

$m|\overline{AC}|^2 = 80$

$m\overline{AC} = \sqrt{80} = 8.94$ feet $\rightarrow (01) mark$

Since $8.94ft < 9ft$, so 9 feet wide table can pass through the rectangular doorway. $\rightarrow (01) mark$

(xii) Consider a parallelogram ABCD.



In right ΔCDA (by Pythagoras Theorem)

$m|\overline{CD}|^2 = m|\overline{AD}|^2 + m|\overline{AC}|^2$ $\rightarrow (01) mark$

$m|\overline{AC}|^2 = m|\overline{CD}|^2 - m|\overline{AD}|^2$

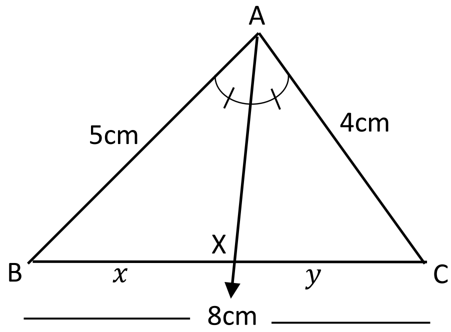
$m|\overline{AC}|^2 = 16^2 - 9^2 = 175$

$m|\overline{AC}| = \sqrt{175} = 13.23m$ $\rightarrow (01) mark$

$Area\ of\ \Delta CDA = \frac{1}{2}(m|\overline{AD}|)(m|\overline{AC}|) = \frac{1}{2}(9)(13.23) = \frac{1}{2}(119.07)$ $\rightarrow (01) mark$

$Area\ of\ parallelogram\ ABCD\ = 2(Area\ of\ \Delta CDA) = 119.07m$ $\rightarrow (01) mark$

$(xiii)$ $x + y = 8 \implies y = 8 - x$ $\rightarrow eqn - I$



$m\overline{BX} : m\overline{CX} = m\overline{AB} : m\overline{AC}$ $\rightarrow (01) mark$

$x : y = 5 : 4$

$4x = 5y$ $\rightarrow (01) mark$

$4x = 5(8 - x)$ From $eqn - I$

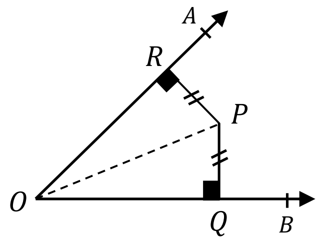
$4x = 40 - 5x$

$x = \frac{40}{9}$ $\rightarrow (01) mark$

Using $eqn - I$

$y = 8 - \frac{40}{9} = \frac{32}{9}$ $\rightarrow (01) mark$

(xiv) **Figure**: $\rightarrow (0.5) mark$



Given: Any point P lies inside $\angle AOB$ such that $\overline{PQ} = \overline{PR}$,

where $\overline{PQ} \perp \overrightarrow{OB}$ and $\overline{PR} \perp \overrightarrow{OA}$. $\rightarrow (0.5) mark$

To Prove: Point P is on the bisector of $\angle AOB$. $\rightarrow (0.5) mark$

Construction: Join P to O. $\rightarrow (0.5) mark$

Proof:

Statements	**Reasons**	
In $\Delta POQ \leftrightarrow \Delta POR$		
$\angle PQO \cong \angle PRO$	Given	$\rightarrow (0.5) mark$
$\overline{PO} \cong \overline{PO}$	Common	$\rightarrow (0.5) mark$
$\overline{PQ} \cong \overline{PR}$	Given	$\rightarrow (0.5) mark$
$\therefore \Delta POQ \cong \Delta POR$	H.S. Postulate	$\rightarrow (0.5) mark$
Hence $\angle POQ \cong \angle POR$	Corresponding angles of congruent triangles	
i.e. P is on the bisector of $\angle AOB$		

SECTION-C

Q 3. $AB = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 9+8 & 21+20 \\ 6+6 & 14+15 \end{bmatrix} = \begin{bmatrix} 17 & 41 \\ 12 & 29 \end{bmatrix}$ $\rightarrow (01)mark$

$|AB| = (17)(29) - (41)(12) = 1$ $\quad Adj(AB) = \begin{bmatrix} 29 & -41 \\ -12 & 17 \end{bmatrix}$ $\rightarrow (0.5 + 0.5)mark$

$(AB)^{-1} = \frac{1}{|AB|}.Adj(AB) = \begin{bmatrix} 29 & -41 \\ -12 & 17 \end{bmatrix}$ $\rightarrow (0.5 + 0.5)mark$

$|B| = (3)(5) - (7)(2) = 1$ $\quad Adj(B) = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$ $\rightarrow (0.5 + 0.5)mark$

$B^{-1} = \frac{1}{|B|}.Adj(B) = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$ $\rightarrow (0.5 + 0.5)mark$

$|A| = (3)(3) - (4)(2) = 1$ $\quad Adj(A) = \begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix}$ $\rightarrow (0.5 + 0.5)mark$

$A^{-1} = \frac{1}{|A|}.Adj(A) = \begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix}$ $\rightarrow (0.5 + 0.5)mark$

$B^{-1}A^{-1} = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 15+14 & -20-21 \\ -6-6 & 8+9 \end{bmatrix} = \begin{bmatrix} 29 & -41 \\ -12 & 17 \end{bmatrix}$ $\rightarrow (01)mark$

***Q*4.** $\frac{x}{x^2 - x - 2} - \frac{1}{x^2 + 5x - 14} - \frac{2}{x^2 + 8x + 7} = \frac{x+3}{x^2 + 5x - 14}$

$x^2 - x - 2 = x^2 - 2x + x - 2 = x(x-2) + 1(x-2) = (x-2)(x+1)$ $\rightarrow (01)mark$

$x^2 + 5x - 14 = x^2 - 2x + 7x - 14 = x(x-2) + 7(x-2) = (x-2)(x+7)$ $\rightarrow (01)mark$

$x^2 + 8x + 7 = x^2 + x + 7x + 7 = x(x+1) + 7(x+1) = (x+1)(x+7)$ $\rightarrow (01)mark$

$x^2 + 5x - 14 = x^2 - 2x + 7x - 14 = x(x-2) + 7(x-2) = (x-2)(x+7)$ $\rightarrow (01)mark$

$\frac{x}{(x-2)(x+1)} - \frac{1}{(x-2)(x+7)} - \frac{2}{(x+1)(x+7)} = \frac{x+3}{(x-2)(x+7)}$

$\frac{x(x+7)-(x+1)-2(x-2)}{(x-2)(x+1)(x+7)} = \frac{x+3}{(x-2)(x+7)}$ $\rightarrow (01)mark$

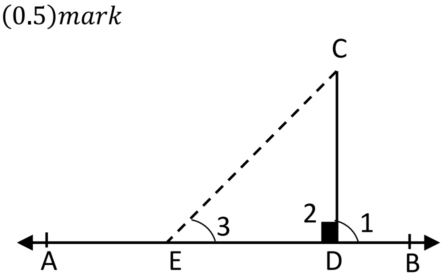
$\frac{x^2+7x-x-1-2x+4}{(x-2)(x+1)(x+7)} = \frac{x+3}{(x-2)(x+7)}$ $\rightarrow (0.5)mark$

$\frac{x^2+4x+3}{(x-2)(x+1)(x+7)} = \frac{x+3}{(x-2)(x+7)}$ $\rightarrow (01)mark$

$\frac{(x+1)(x+3)}{(x-2)(x+1)(x+7)} = \frac{x+3}{(x-2)(x+7)}$ $\rightarrow (01)mark$

$\frac{x+3}{(x-2)(x+7)} = \frac{x+3}{(x-2)(x+7)}$ $\rightarrow (0.5)mark$

Q5. **Figure**: $\rightarrow (01)mark$



Given: A point C not lying on $\overleftrightarrow{AB}$. A point D lying on $\overleftrightarrow{AB}$ such that $\overline{CD} \perp \overleftrightarrow{AB}$. $\rightarrow (01)mark$

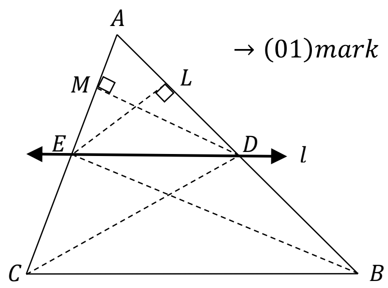
To Prove: $\overline{CD}$ is the shortest distance from C to $\overleftrightarrow{AB}$. $\rightarrow (01)mark$

Construction: Take a point E on $\overleftrightarrow{AB}$. Join C to E to form a ΔCDE. $\rightarrow (01)mark$

Proof:

Statements	**Reasons**	
In ΔCDE $m\angle 1 > m\angle 3$ $\rightarrow (i)$	An exterior angle of a triangle is greater than non-adjacent interior angle	$\rightarrow (01)mark$
$m\angle 1 = m\angle 2$ $\rightarrow (ii)$	Supplement of right angle	$\rightarrow (01)mark$
$m\angle 2 > m\angle 3$	from (i) & (ii)	$\rightarrow (0.5)mark$
$m\angle 3 < m\angle 2$	If $a > b$ then $b < a$	
$m\overline{CD} < m\overline{CE}$	Opposite side of smaller angle	$\rightarrow (01)mark$
But E is any point on AB		
Hence $\overline{CD}$ is the shortest distance from C to $\overleftrightarrow{AB}$		$\rightarrow (0.5)mark$

Q6. Figure: $\rightarrow (01)mark$



Given: In ΔABC, line l is intersecting sides $\overline{AC}$ and $\overline{AB}$ at points E and D respectively such that $\overline{ED} \parallel \overline{CB}$. $\rightarrow (01)mark$

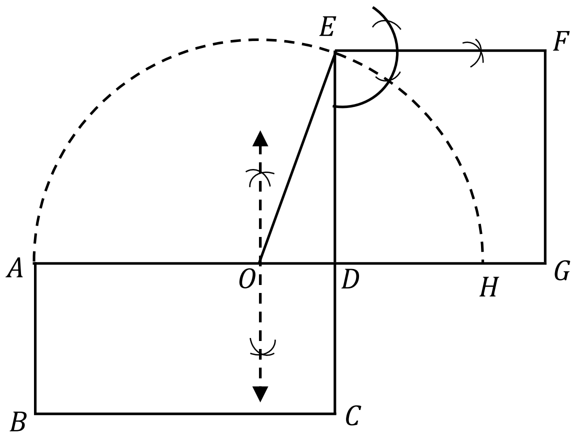
To Prove: m$\overline{AD}$: m$\overline{DB}$ = m$\overline{AE}$: m$\overline{EC}$ $\rightarrow (01)mark$

Construction: Join B to E; C to D. Draw $\overline{DM} \perp \overline{AC}$ and $\overline{EL} \perp \overline{AB}$. $\rightarrow (01)mark$

Proof:

Statements	**Reasons**	
In triangles BED and AED, $\overline{EL}$ is the common perpendicular.		
$\therefore Area\ of \Delta BED = \frac{1}{2}(m\overline{BD})(m\overline{EL})$ $\rightarrow (i)$	Area of a Δ = $\frac{1}{2}$ (base) (height)	$\rightarrow (0.5)mark$
$\therefore Area\ of\ \Delta AED = \frac{1}{2}(m\overline{AD})(m\overline{EL})$ $\rightarrow (ii)$	Area of a Δ = $\frac{1}{2}$ (base) (height)	$\rightarrow (0.5)mark$
$\Rightarrow \frac{Area\ of\ \Delta BED}{Area\ of\ \Delta AED} = \frac{m\overline{DB}}{m\overline{AD}}$ $\rightarrow (iii)$	Dividing (i) by (ii)	$\rightarrow (0.5)mark$
$\Rightarrow \frac{Area\ of\ \Delta CDE}{Area\ of\ \Delta ADE} = \frac{m\overline{EC}}{m\overline{AE}}$ $\rightarrow (iv)$	similarly	$\rightarrow (0.5)mark$
But Area of $\Delta BED \cong Area\ of\ \Delta CDE$	Areas of triangle with common base and same altitudes are equal. Given that $\overline{ED} \parallel \overline{CB}$. So altitudes are equal.	$\rightarrow (01)mark$
$\frac{m\overline{DB}}{m\overline{AD}} = \frac{m\overline{EC}}{m\overline{AE}}$	From (iii) and (iv)	$\rightarrow (0.5)mark$
$\frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EC}}$	On taking reciprocals	$\rightarrow (0.5)mark$
$m\overline{AD} : m\overline{DB} = m\overline{AE} : m\overline{EC}$		

Q7. (a) **Construction Steps**



(i) Construct a 4 by 2 rectangle. $\rightarrow (01)mark$

(ii) Produce $\overline{AD}$ to H making $m\overline{DH} = m\overline{CD}$.

(iii) Bisect $\overline{AH}$ at O. $\rightarrow (01)mark$

(iv) With centre O and radius $\overline{OA}$ describe a semi-circle. $\rightarrow (01)mark$

(v) Produce $\overline{CD}$ to meet the semi-circle in E.

(vi) On $\overline{DE}$ as a side construct a square DGFE (the required one). $\rightarrow (01)mark$

(b) $m\overline{DG} = m\overline{GF} = m\overline{FE} = m\overline{DE} = 2.8cm$ $\rightarrow (01)mark$

Area of Square $DGFE = (2.8)(2.8) = 7.84cm^2$ $\rightarrow (01)mark$

(c) Area of Rectangle $ABCD = (4)(2) = 8cm^2$ $\rightarrow (01)mark$

$Area\ of\ Square\ DGFE \approx Area\ of\ Rectangle\ ABCD$ $\rightarrow (01)mark$