SOLUTION QUESTION MODEL PAPER (3rd Set) SSC-II
MATHEMATICS

SECTION-A

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	B	A	B	B	C	A	C	C	A	A	D	B	C	B

SECTION-B

Question 2

(i) $\frac{2x+1}{x+2} - \frac{2x+4}{2x+8} = 0$

$(2x+1)(2x+8) - (2x+4)(x+2) = 0$

$4x^2 + 16x + 2x + 8 - 2x^2 - 4x - 4x - 8 = 0$

$x^2 + 5x + 0 = 0$

Applying the quadratic formula for x

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ Where $a = 1, b = 5, c = 0$

$x = \frac{-5 \pm \sqrt{25-0}}{2}$

$x = \frac{-5 \pm 5}{2}$

$x = -5$ or $x = 0$

Solution Set= $\{-5, 0\}$

(ii) $3.3^{2x+1} - 10.3^x + 1 = 0$

$3.3.3^{2x} - 10.3^x + 1 = 0$

$9(3^x)^2 - 10(3^x) + 1 = 0$

Let $3^x = y$ $\rightarrow eqn - I$

$9y^2 - 10y + 1 = 0$

$9y^2 - 9y - y + 1 = 0$

$9y(y-1) - 1(y-1) = 0$

$(y-1)(9y-1) = 0$

$y - 1 = 0$ or $9y - 1 = 0$

$y = 1$ or $y = \frac{1}{9}$

Putting the value of y in $eqn - I$

$3^x = 1$ or $3^x = \frac{1}{9}$

$3^x = 3^0$ or $3^x = 3^{-2}$

$x = 0$ or $x = -2$

(iii) If $\theta, \emptyset$ are the roots of $y^2 - 7y + 9 = 0$, then

$$Sum\ of\ the\ roots\ =\ \theta + \emptyset = -\frac{\text{coeff of } y}{\text{coeff of } y^2} = -\frac{-7}{1} = 7$$

$$Product\ of\ the\ roots =\ \theta\emptyset = \frac{\text{constt term}}{\text{coeff of } y^2} = \frac{9}{1} = 9$$

If roots of the required equation are $2\theta, 2\emptyset$ then,

Sum of the roots: $S = 2\theta + 2\emptyset = 2(\theta + \emptyset) = 2(7) = 14$

Product of the roots: $P = (2\theta)(2\emptyset) = 4(\theta\emptyset) = 4(9) = 36$

Required quadratic equation: $y^2 + Sy + P = 0$

$$y^2 + 14y + 36 = 0$$

(iv) If xcm be the breadth of the rectangle, then it's length = $(x + 5)cm$

Area of the rectangle: $x(x + 5) = 50$

$$x^2 + 5x - 50 = 0$$

$$(x - 5)(x + 10) = 0$$

$$x - 5 = 0\ or\ x + 10 = 0$$

$$x = 5\ or\ x = -10$$

Breadth: $x = 5cm$ (neglecting the negative value)

Length: $x + 5 = 5 + 5 = 10cm$

(v) If a be the fourth proportional, then

$(x^3 - y^3) : (x^2 - y^2) :: (y^2 + 2xy + y^2) : a$

Product of Extremes = Product of Means

$$(a)(x^3 - y^3) = (x^2 - y^2)(y^2 + 2xy + y^2)$$

$$a = \frac{(x^2 - y^2)(2y^2 + 2xy)}{(x^3 - y^3)}$$

$$a = \frac{(x - y)(x + y)(x + y)2y}{(x - y)(x^2 + xy + y^2)}$$

$$a = \frac{2y(x + y)^2}{(x^2 + xy + y^2)}$$

(vi) $I \propto E\ \ and\ \ I \propto \frac{1}{R} \quad \Rightarrow I \propto \frac{E}{R} \quad \Rightarrow I = \frac{kE}{R} \quad \rightarrow eqn - I$

For $I = 32\ amp,\ E = 1280\ volts\ and\ R = 80\ ohm$

$$32 = \frac{k(1280)}{80}$$

$$k = \frac{(80)(32)}{1280} = 2$$

Putting k's value in $eqn - I$

$$I = \frac{2E}{R}$$

When $E = 1500\ volts\ and\ R = 180\ ohm$

$$I = \frac{2(1500)}{180} = \frac{50}{3}amp$$

(vii) $\dfrac{4x+2}{2(x-1)(x^2+1)^2} = \dfrac{2(2x+1)}{2(x-1)(x^2+1)^2}$

$$\frac{(2x+1)}{(x-1)(x^2+1)^2} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2} \qquad \rightarrow eqn - I$$

Multiplying both sides by $(x-1)(x^2+1)^2$

$$2x+1 = A(x^2+1)^2 + (Bx+C)(x-1)(x^2+1) + (Dx+E)(x-1) \qquad \rightarrow eqn - II$$

For A put $x-1=0$ or $x=1$ in $eqn - I$

$$2(1)+1 = A(1^2+1)^2 + 0 + 0 \qquad \Longrightarrow \boldsymbol{A} = \frac{3}{4}$$

Simplifying $eqn - II$

$$2x+1 = A(x^4+2x^2+1) + B(x^4-x^3+x^2-x) + C(x^3-x^2+x-1) + D(x^2-x) + E(x-1)$$

Equating the coefficients of like powers of x^4, x^3, x^2, x

Coeff of x^4: $0 = A + B \qquad \Longrightarrow \boldsymbol{B} = -A = -\frac{3}{4}$

Coeff of x^3: $0 = -B + C \qquad \Longrightarrow \boldsymbol{C} = B = -\frac{3}{4}$

Coeff of x^2: $0 = 2A + B - C + D \qquad \Longrightarrow \boldsymbol{D} = C - B - 2A = -\frac{3}{4} + \frac{3}{4} - 2\left(\frac{3}{4}\right) = -\frac{3}{2}$

Coeff of x: $2 = -B + C - D + E \qquad \Longrightarrow \boldsymbol{E} = D - C + B + 2 = -\frac{3}{2} + \frac{3}{4} - \frac{3}{4} + 2 = \frac{1}{2}$

Substituting the values of A, B, C, D and E in $eqn - I$

$$\frac{2x+1}{(x-1)(x^2+1)} = \frac{3}{4(x-1)} - \frac{3(x+1)}{4(x^2+1)} - \frac{(3x-1)}{2(x^2+1)^2}$$

$(viii)$ $U = \{1,2,3,\dots,20\}$, $A = \{1,2,3,\dots,10\}$, $B = \{2,4,6,8,10,12,14,16\}$

$A \cup B = \{1,2,3,4,5,6,7,8,9,10,12,14,16\}$, $A \cap B = \{2,4,6,8,10\}$

$(A \cap B)^c = \cup - (A \cap B) = \{1,3,5,7,9,11,12,13,14,15,16,17,18,19,20\}$

$(A \cup B) - (A \cap B)^c = \{1,2,3,,,10,12,14,16\} - \{1,3,5,7,9,11,12,13,14,,,19,20\} = \{2,4,6,8,10\}$

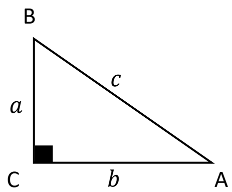
(ix)

x	90	80	70	90	$\sum x = 330$
x^2	8100	6400	4900	8100	$\sum x^2 = 27500$

Number of values: $n = 4$

Variance: $S^2 = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2 = \frac{27500}{4} - \left(\frac{330}{4}\right)^2 = 68.75$

Standard Deviation: $S = \sqrt{68.75} = 8.29$



(x) Consider a right triangle ABC with $m\angle C = 90°$

$tan\,\theta = \frac{a}{b} = \frac{\sqrt{7}}{2} \qquad \Longrightarrow a = \sqrt{7}, b = 2$

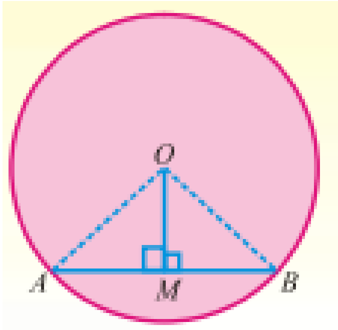
By Pythagoras Theorem

$c^2 = a^2 + b^2 \qquad c = \sqrt{(\sqrt{7})^2 + (2)^2} = \sqrt{11}$

$sin\,\theta = \frac{a}{c} = \frac{\sqrt{7}}{\sqrt{11}} \qquad cos\,\theta = \frac{b}{c} = \frac{2}{\sqrt{11}} \qquad tan\,\theta = \frac{a}{b} = \frac{\sqrt{7}}{2}$

$csc\,\theta = \frac{c}{a} = \frac{\sqrt{11}}{\sqrt{7}} \qquad sec\,\theta = \frac{\sqrt{11}}{2} \qquad cot\,\theta = \frac{b}{a} = \frac{2}{\sqrt{7}}$

(xi) **Figure**:



Given*:* $\overline{AB}$ is the chord of a circle with centre at O so that $\overline{OM} \perp$ chord $\overline{AB}$.

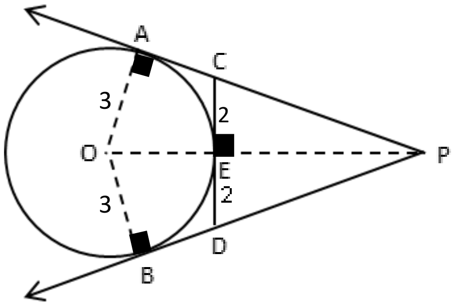
To prove*:* M is the midpoint of chord $\overline{AB}$ i.e. $m\overline{AM} = m\overline{BM}$.

Construction*:* Join O to A and B.

Proof:

Statements	Reasons
In $\angle rt\ \Delta^s\ OAM \leftrightarrow OBM$	
$m\angle OMA = m\angle OMB = 90°$	Given
$Hyp\ m\overline{OA} = Hyp\ m\overline{OB}$	Radii of the same circle
$m\overline{OM} = m\overline{OM}$	Common
$\therefore \Delta OAM \cong \Delta OBM$	In $\angle rt\ \Delta^s \quad H.S \cong H.S$
Hence, $m\overline{AB} = m\overline{BM}$	Corresponding sides of congruent triangles.
$\Longrightarrow \overline{OM}$ bisects the chord $\overline{AB}$.	

(xii) In the figure given that



$m\overline{CE} = m\overline{DE} = 2cm\ , \quad m\overline{OA} = m\overline{OB} = m\overline{OE} = 3cm$

$m\overline{PA} = m\overline{BP} \ = \ 8\ cm$

In an Isosceles ΔPCD

$m\overline{PC} = m\overline{PD} \qquad m\overline{PC} + m\overline{PD} = ?$

In rt.ΔPOA, (Pythagoras Theorem)

$$(m\overline{OP})^2 = (m\overline{OA})^2 + (m\overline{AP})^2 = 3^2 + 8^2 = 73$$

$$m\overline{OP} = \sqrt{73} = 8.54$$

$$\overline{PE} = \overline{OP} - \overline{OE} = 8.54 - 3 = 5.54$$

In rt.ΔPCE, (Pythagoras Theorem)

$$(m\overline{PC})^2 = (m\overline{CE})^2 + (m\overline{PE})^2 = 2^2 + 5.54^2 = 34.69$$

$$m\overline{PC} = \sqrt{34.69} = 5.89cm = m\overline{PD}$$

$$m\overline{PC} + m\overline{PD} = 5.89 + 5.89 = 11.78cm$$

$(xiii)$ $Given\ in \triangle ABC, \qquad m\angle CAD = a = 30°,\ \ m\angle ACB = d = 45°$

$O\ is\ the\ center\ of\ the\ circle$

$from\ figure\ \widehat{ADC}\ is\ a\ semi-circle$

$m\angle ADC = f = 90°$

$e = 180 - (a + f) = 180 - (30 + 90) = 60°$

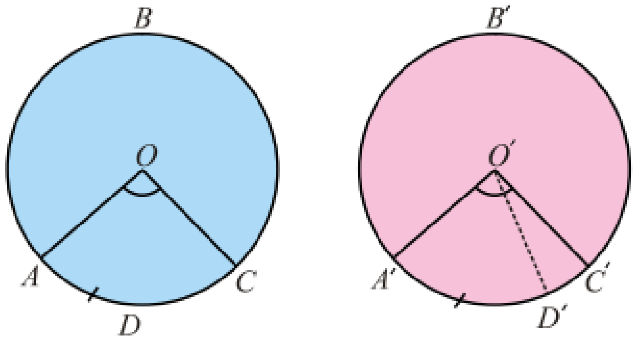
$Again\ from\ fig, \widehat{ABC}\ is\ a\ semi-circle$

$m\angle ABC = c = 90°$

$b = 180° - (d + c) = 180° - (45° + 90°) = 45°$

(xiv)

Figure:



Given: ABC and A′B′C′ are two congruent circles with centers O and O' respectively so that $\overline{AC} = \overline{A'C'}$

To prove: $\angle AOC \cong \angle A'O'C$

Construction: Let if possible $m\angle AOC \neq m\angle A'O'C'$ then consider $\angle AOC \cong \angle A'O'D'$

Proof:

Statements	Reasons
$\angle AOC = \angle A'O'D'$	Construction
$\therefore \quad \widehat{AC} \cong \widehat{A'D'} \qquad eqn-I$	Areas subtended by equal central angles in congruent circles
$m\overline{AC} = m\overline{A'D'} \qquad eqn-II$	If two arcs of a circle are congruent then corresponding chords are equal
But $m\,\overline{AC} = m\,\overline{A'C'} \qquad eqn-III$	Given
$\therefore m\overline{A'C'} = m\overline{A'D'}$	Using $eqns-II\ \&\ III$
Which is only possible, if C' coincides with D'.	
Hence $m\angle A'O'C' = m\angle A'O'D' \qquad eqn-IV$	
But $m\angle AOC = m\angle A'O'D' \qquad eqn-V$	Construction
$\Rightarrow \angle AOC = \angle A'O'C'$	Using $eqns-IV\ \&\ V$

SECTION − C

***Q*3.** $x^4 - 4x^3 - 3x^2 + 4x + 1 = 0$

$$x^2 - 4x - 3 + \frac{4}{x} + \frac{1}{x^2} = 0 \qquad (Divided\ by\ x^2)$$

$$x^2 + \frac{1}{x^2} - 4x + \frac{4}{x} - 3 = 0$$

$$\left(x^2 + \frac{1}{x^2}\right) - 4\left(x - \frac{1}{x}\right) - 3 = 0$$

$$Let \quad x - \frac{1}{x} = y \quad \Rightarrow \quad \left(x^2 + \frac{1}{x^2}\right) = y^2 + 2$$

$$(y^2 + 2) - 4y - 3 = 0$$

$$y^2 - 4y - 1 = 0$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad with\ a = 1, b = -4, c = -1$$

$$y = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-1)}}{2(1)}$$

$$y = \frac{4 \pm \sqrt{16+4}}{2} = \frac{4 \pm \sqrt{20}}{2} = \frac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$$

$x - \frac{1}{x} = 2 \pm \sqrt{5}$ $\quad \Rightarrow x^2 - (2 \pm \sqrt{5})\,x - 1 = 0$

$x^2 - (2+\sqrt{5})\,x - 1 = 0$; $x^2 - (2-\sqrt{5})\,x - 1 = 0$

$$x = \frac{(2+\sqrt{5}) + \sqrt{(2+\sqrt{5})^2 - 4(1)(-1)}}{2(1)} \quad ; \quad x = \frac{(2-\sqrt{5}) - \sqrt{(2-\sqrt{5})^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{(2+\sqrt{5}) + \sqrt{13+4\sqrt{5}}}{2} \quad ; \quad x = \frac{(2-\sqrt{5}) - \sqrt{13-4\sqrt{5}}}{2}$$

Solution set $= \left\{\frac{(2+\sqrt{5})+\sqrt{13+4\sqrt{5}}}{2}, \frac{(2-\sqrt{5})-\sqrt{13-4\sqrt{5}}}{2}\right\}$

Q4 $U = \{5,6,7,8,9,\ldots,20\}$, $A = \{6,8,10,\ldots,20\}$, $B = \{5,7,11,13,17,19\}$

(i) $A \cup B = \{5,6,7,8,10,11,12,13,14,16,17,18,19,20\}$

$(A \cup B)^c = U - (A \cup B) = \{9,15\}$ $\quad \rightarrow eqn - I$

$A^c = U - A = \{5,7,9,11,13,15,17,19\}$, $\quad B^c = U - B = \{6,8,9,10,12,14,15,16,18\}$

$A^c \cap B^c = \{9,15\}$ $\quad \rightarrow eqn - II$

From $eqns - I$ & II

$(A \cup B)^c = A^c \cap B^c$

(ii) $A \cap B = \emptyset$

$(A \cap B)^c = U - A \cap B = \{5,6,7,8,9,\ldots,20\}$ $\quad \rightarrow eqn - III$

$A^c \cup B^c = \{5,6,7,8,9,\ldots,20\}$ $\quad \rightarrow eqn - IV$

From $eqns - III$ & IV

$(A \cap B)^c = A^c \cup B^c$

Q5.

Class Interval	*Mid Value* (x)	f	$log\ x$	$f\ log\ x$	f/x
0 —10	05	3	0.6989	2.0969	0.6
10 — 20	15	4	1.17609	4.7043	0.266
20 — 30	25	5	1.3979	6.9897	0.2
30 — 40	35	6	1.54406	9.26440	0.1714
40 — 50	45	7	1.65321	11. 57240	0.1555
		$\sum = 25$		$\sum = 34.62778$	$\sum = 1.3929$

Geometric Mean= $Antilog\left[\frac{\sum(f \log x)}{\sum f}\right] = Antilog\left(\frac{34.62778}{25}\right) = 24.273$

Harmonic Mean $= \frac{\sum f}{\sum\left(\frac{f}{x}\right)} = \frac{25}{1.3929} = 17.95$

Q 6. **Construction Steps:**

i. Draw a circle of radius 5 cm.

ii. Draw diameter $\overline{AD}$

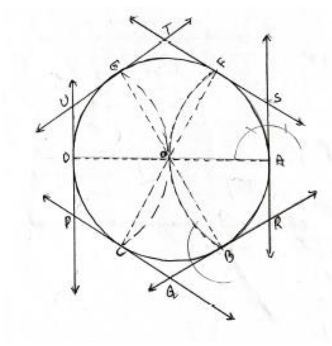
iii. From point A draw an arc of radius $\overline{AO}$, which cuts the circle at points B and F.

iv. Join B with O and extend it to meet the circle at E.

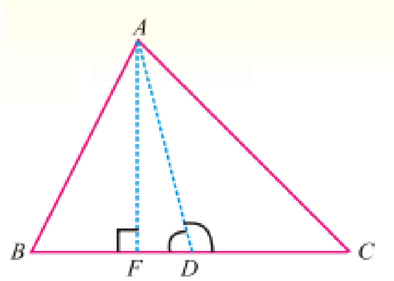
v. Join F with O and extend it to meet the circle at C.

vi. Draw tangents to the circle at points A, B, C, D, E and F intersecting one another at points P, Q, R, S, T, U respectively.

Thus, $PQRSTU$ is the circumscribed regular hexagon (figure)



Q7 **Figure:**



Given: In ΔABC, the median $\overline{\text{AD}}$ bisects $\overline{\text{BC}}$.

i.e. m $\overline{\text{BD}}$ = $m\overline{\text{CD}}$.

To prove: $(AB)^2 + (AC)^2 = 2(BD)^2 + 2(AD)^2$

Construction: Draw $\overline{\text{AF}} \perp \overline{\text{BC}}$

Proof: (04)

Statements	Reasons
In ΔADB $\angle ADB$ is acute at D	
$\therefore (AB)^2 = (BD)^2 + (AD)^2 - 2\, m\overline{BD}\, .m\overline{FD}$ (i)	In any triangle, the square on the side opposite to acute angle is equal to the sum of squares on the sides containing that acute angle diminished by twice the rectangle contained by one of those sides and projection on it of the other.
In ΔADC $\angle ADC$ is obtuse at D	
$\therefore (AC)^2 = (CD)^2 + (AD)^2 + 2\, \text{m}\overline{\text{CD}}\, .m\overline{\text{FD}}$	In an obtuse angled triangle, the square on the side opposite to obtuse angle is equal to the sum of squares on the sides containing that obtuse angle diminished by twice the rectangle contained by one of those sides and projection on it of the other.
$(AC)^2 = (BD)^2 + (AD)^2 + 2\, m\overline{\text{BD}}\, .m\overline{\text{FD}}$ (ii)	$(BD)^2 = (CD)^2$
$(AB)^2 + (AC)^2 = 2(BD)^2 + 2(AD)^2$	Adding (i) and (ii)